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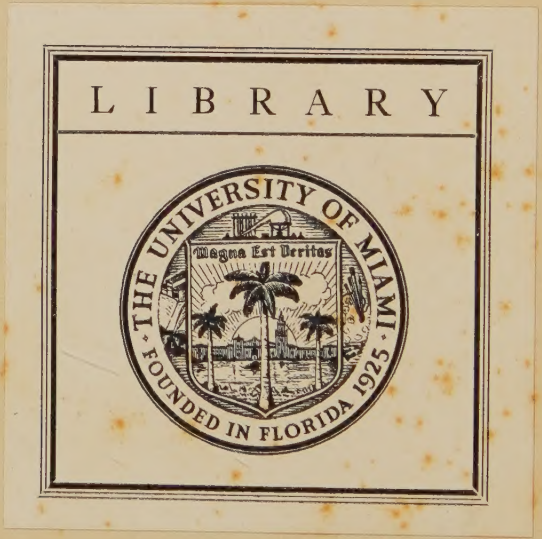

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ELEMENTARY STATISTICS

By

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With a Preface by

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PREFACE

This book is a primer on elementary statistics. It presents the first steps in statistical method without confusing the students with the more advanced processes and theoretical considerations. There is an easy sequence of steps, each built on the preceding material. The book contains explanatory definitions in simple language of all processes, explanatory problems of each type fully worked out and analyzed, practice problems presented in the order of difficulty, explanatory charts, page references to other books for each important process, answers to problems, and numerous suggestions for short cuts in time saving. There is an almost complete absence of theoretical matter.

A person who is sufficiently interested can learn the processes by following the steps presented, almost, if not quite, without instruction.

The need for such a guide is obvious to the reader of current educational journals. More and more the articles appearing in journals on education contain statistical material. A knowledge of elementary statistics is essential for the intelligent reading and interpretation of current educational literature. For those who wish to become expert statisticians, and who desire to make extensive studies involving a statistical technique, more advanced courses in statistics will be required. But for those who are entering upon the study of education in normal schools, teachers' colleges, and colleges of education, and for those who are engaged

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in teaching, a familiarity with the statistics of this book is essential. It presents the tools of the trade, the instruments of learning, and the technique of modern scientific investigation in their simplest and most elementary form.

L. D. COFFMAN.

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INTRODUCTION

MEANING AND SCOPE OF ELEMENTARY STATISTICS

Meaning. — Statistical methods are ways of stating facts in the form of numbers. Such methods, because of their objective nature, have an advantage over other methods of evaluation, because they are impersonal and exact. Feelings and preferences may be eliminated when we deal with facts numerically.

The uses of statistics. — A good example of the practical use of statistics is the report of the United States Census. Here we have a vast amount of impartially compiled data expressed entirely in numbers, usually carried out to one decimal place, after having been treated according to correct statistical procedure. We can have only the highest regard for such reports, because of the sense of security we obtain from their use.

In nearly every line of human endeavor there is increasing need of statistical methods. This is a scientific age. The development of any science is dependent to a large degree upon the adaptability of its data to measurement. Astronomy, physics, engineering, and chemistry have become "exact" sciences for this reason; economics, biology, sociology, psychology, and education are following along similar lines. The more rapidly we advance from speculation and guessing, the greater the need for precise and convenient ways of saying what we mean. The chief function of statistical methods is to fill this need.

How technical must we be? — One of the characteristics of statistical methods, as usually presented, is that they are highly technical and involved. This is not surprising, inasmuch as they have been developed largely by mathematicians, and their appeal is often limited to students of advanced courses. Beginning students in statistical methods frequently fear the competition with advanced students of mathematical bent, and their fears are apparently justified when the work stresses the theoretical aspects of the problem. While it might be observed that other college and university courses are similarly affected, it is especially unfortunate in the case of statistics, which have become fundamental to the pursuit of learning in many branches.

One can hardly follow the leading articles in current professional magazines without at least an elementary knowledge of statistical terms and procedure. The following quotation from an educational magazine* will serve to illustrate this point:

TABLE IV. DATA ON PER CENT OF PUPIL FAILURES. BY SCHOOLS.

	1925	1926
Mean	10.3	7.8
Standard Deviation	5.3	4.7
Range:		
Maximum	35.2	19.6
Minimum	0.4	0.0

"Two significant interpretations are possible from Table IV: (1) Fewer pupils were failed this year than last year,

* Journal of Educational Research, November, 1926.

for the 1926 mean per cent is 2.5 smaller than the 1925 mean. Roughly, this indicates that nearly eight thousand fewer elementary-school children were failed. (2) The amount of failure among the schools was more uniform this year than last. In 1926 the schools clustered more closely around the central tendency than in 1925. This is shown by the smaller standard deviation and the contracted range . . . "

The foregoing quotation is a part of an interesting article on the causes of school failures. But important as the subject may be, it is likely to pass "over the head" of many a school superintendent, principal, and teacher, because they do not understand the terms "range," "mean," and "standard deviation" in their statistical significance. But these terms are not a part of advanced statistics. They may be learned easily, with the aid of this book, even without a teacher if need be, and without the use of knowledge any more technical than simple arithmetic.

The task of this book. — This book has for its task the presentation of the simpler and necessary statistical methods, in the form of graded problems which illustrate the procedure without involving extensive computation, or any knowledge of higher mathematics. Beginning with the simplest principles of statistical usage, we shall take up in turn the methods which will serve as working tools for anyone who has occasion to deal with numerically expressed facts. Each new type of problem is based on the processes used in the problems preceding it. It is advisable, therefore, for the beginner to give some attention to each section, in its turn, rather than attempt to learn at once any of the measures presented in the latter part of the book.

The further study of statistics. — Those who wish to pursue the interesting study of statistical methods,

and their varied applications, will find the appended list of references helpful. It is by no means exhaustive, but it includes some of the leading works, and most of these books contain additional references. For the benefit of students who wish to check on specific methods, or who desire further enlightenment on the subjects treated in this book, specific page references are given at the close of the principal chapters.

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PART I

THE STATISTICAL ARRANGEMENT OF NUMBERS

ELEMENTARY STATISTICS

CHAPTER I

HOW NUMBERS SHOULD READ

Explanation. — In most forms of statistical work it is necessary to have the numbers with which we deal arranged in convenient and logical order.

Numbers may be arranged in *horizontal* or *vertical* rows. Horizontally, they should read, in order of increasing value, from left to right.

ILLUSTRATIVE PROBLEM. — Arrange the following numbers horizontally in statistical order:

83, 67, 63, 68, 75, 70, 92, 98, 88, 83, 75, 62.

Solution:

62, 63, 67, 68, 70, 75, 75, 83, 83, 88, 92, 98.

Vertically, numbers should read from the bottom upward; so that the highest number will be at the top, the lowest at the bottom, and each number in its respective place

ILLUSTRATIVE PROBLEM. — Arrange vertically:

80, 72, 71, 90,

73, 87, 93, 75,

80, 86, 90, 84,

Solution:

93
90
90
87
86
84
80
80
75
73
72
71

PROBLEMS TO SOLVE

Arrange horizontally:

1. 75, 82, 64, 74, 70, 96, 90, 63, 71, 75, 66, 92.
2. 80, 70, 54, 63, 58, 52, 91, 88, 84, 70, 75, 63, 58.
3. 43, 48, 55, 43, 47, 35, 64, 65, 60, 44, 45, 37, 56, 65.
4. 17, 15, 18, 17, 17, 19, 18, 24, 32, 15, 13, 30, 18, 17, 19, 26.
5. 156, 142, 158, 147, 140, 132, 137, 150, 146, 158, 100, 109.
6. 3, 7, 4, 8, 7, 6, 6, 8, 5, 4, 3, 4, 1, 7, 0, 8, 9, 7, 3, 8, 9, 6.
7. 27, 22, 20, 25, 21, 20, 18, 25, 22, 25, 27, 20, 21, 18, 25.
8. 16, 15, 12, 16, 15, 14, 15, 16, 17, 19, 16, 15, 18, 16, 17, 19.
9. 8, 5, 7, 6, 0, 3, 0, 7, 3, 3, 5, 8, 7, 0, 3, 0, 5, 0, 1, 7, 3.
10. 96, 98, 54, 55, 63, 68, 92, 65, 70, 83, 73, 75, 60, 74, 75, 90, 84, 80, 66, 88, 70, 91, 90, 87, 65, 58.
11. 47, 53, 48, 50, 45, 50, 58, 56, 41, 48, 47, 54, 53, 46, 50, 49, 50, 43, 40, 56, 38, 58, 60, 51, 54, 37.
12. 9, 8, 4, 5, 7, 8, 9, 7, 6, 3, 3, 3, 4, 8, 7, 5, 2, 1, 5, 0, 3, 8, 4, 2, 9, 7, 5, 6, 4, 3, 8, 5, 2, 9, 5, 7, 3, 8.
13. 148, 156, 150, 147, 155, 168, 173, 170, 175, 160, 150, 143, 152, 140, 130, 173, 143, 158, 166, 170, 160, 154, 163.
14. 50, 80, 70, 40, 30, 80, 70, 80, 60, 70, 50, 40, 50, 40, 80, 70, 30, 80, 70, 80, 50, 40, 20, 80, 60, 50, 60.
15. 150, 147, 160, 180, 100, 75, 83, 54, 68, 70, 90, 70, 90, 98, 97, 88, 96, 104, 83, 110, 118, 95, 101, 115, 113, 100, 99.

Arrange vertically:

16. 86, 80, 93, 83, 68, 90, 88, 79, 70, 75.
17. 15, 12, 7, 2, 16, 10, 18, 16, 8, 7, 1, 4, 15.
18. 37, 43, 40, 27, 30, 18, 42, 35, 25, 20, 39, 20, 29, 32.
19. 92, 81, 85, 94, 90, 86, 82, 70, 88, 78, 83, 90, 88.
20. 87, 80, 85, 82, 80, 86, 85, 89, 80, 75, 92, 86, 84.
21. 64, 66, 63, 62, 60, 61, 64, 66, 61, 68, 69, 60, 70, 63.
22. 114, 105, 103, 113, 107, 102, 114, 116, 109, 105, 108, 117, 106, 101.
23. 29, 25, 18, 28, 20, 19, 22, 16, 8, 12, 29, 13, 16, 27, 21.

24. 8, 3, 6, 11, 18, 22, 1, 12, 17, 21, 4, 7, 19, 15, 2, 9, 16, 20, 14, 5, 10, 13.

25. 23, 18, 46, 58, 27, 45, 32, 47, 58, 27, 45, 18, 16, 29, 32.

26. 4, 1, 3, 5, 8, 1, 2, 5, 6, 4, 0, 1, 2, 7, 0, 2, 3, 0, 3, 5, 7, 0, 1, 4, 6.

27. 95, 98, 94, 97, 93, 96, 95, 98, 94, 93, 97, 95, 98, 97, 96, 92.

28. 135, 142, 158, 146, 157, 152, 145, 150, 140, 137, 152, 163, 104.

29. 95, 92, 98, 93, 90, 97, 90, 98, 100, 96, 91, 90, 82, 80, 75, 90, 84, 88, 80, 75, 90, 87, 80, 100, 93.

30. 62, 68, 54, 61, 70, 62, 63, 62, 68, 64, 68, 70, 68, 73, 70, 68, 61, 54, 68, 64, 62, 70, 69, 70, 68.

31. 5, 40, 5, 40, 10, 35, 30, 5, 25, 15, 30, 45, 20, 10, 25, 15, 15, 30, 35, 40, 35, 20, 10, 20, 15, 40, 35, 5, 45.

32. 300, 500, 300, 700, 100, 500, 1000, 400, 200, 500, 200, 600, 300, 700, 500, 900, 200, 800, 600, 900, 1000, 400, 500, 100, 900, 400.

33. 43, 4, 11, 46, 1, 16, 36, 13, 2, 39, 5, 2, 14, 40, 32, 3, 12, 40, 37, 37, 6, 12, 38, 48, 7, 40, 8, 47, 9, 12, 38, 29, 10, 35, 47, 12, 50, 15, 13, 45, 41, 30.

34. 87, 43, 127, 145, 100, 89, 47, 36, 130, 157, 110, 135, 150, 85, 67, 38, 159, 175, 151, 36, 89, 115, 147, 148, 45, 38, 56, 139, 125, 99, 73, 140, 96, 73, 55.

35. 40, 75, 76, 32, 125, 38, 142, 50, 67, 58, 50, 100, 85, 70, 100, 82, 53, 16, 29, 87, 82, 99, 88, 100, 102, 96, 51, 83, 88, 120, 88, 136, 92, 45, 83, 45, 50, 90, 76, 85, 77, 92, 95, 38, 142 159, 30, 83, 15, 72, 96, 95, 70, 43, 80, 27, 88, 70, 102, 99, 70, 143, 161, 100, 98, 53, 62, 105, 96, 63, 68, 50, 60.

36. 45.0, 38.3, 75.2, 38.5, 70.1, 35.0, 78.3, 42.9, 43.7, 38.2, 39.5, 40.0, 63.7, 46.5, 38.3, 70.2, 29.7, 36.0, 45.2, 69.1, 72.8, 37.3, 45.1, 50.2.

CHAPTER II

FREQUENCY DISTRIBUTION

Explanation. — It is often desirable to condense statistical data, especially when the number of cases is large, or when the numbers are duplicated so often that it is laborious to rewrite them each time they occur.

The following illustrative problem shows how we may *distribute* the ages of a group of children, by first writing down each age that occurs within the range covered, and then *tallying* the ages as we check them off the original records from which they are obtained.

ILLUSTRATIVE PROBLEM. — Arrange the following numbers in a frequency distribution:

9, 12, 7, 10, 8, 11, 6, 11, 8, 9, 12, 8, 7, 10, 9, 7, 9, 8, 9, 7, 10, 9, 10, 8, 9.

Solution:

		AGE	f
12 //		12	2
11 //		11	2
10 ////	which may be	10	4
9 ####//	re-written	9	7
8 ####	thus →	8	5
7 ////		7	4
6 /		6	1
		N	25

In tallying, we simply make a mark after each number, as it is read off the original list, indicating every fifth occurrence with a cross-tally. In the resulting frequency table, shown at the right, "f" means *frequency*, or the number of times each number occurred; "N" means *number of cases*.

ILLUSTRATIVE PROBLEM. — Arrange the following test scores in a frequency distribution:

92, 97, 93, 90, 93, 96, 99, 95, 91, 93, 92, 96, 94, 93, 95, 97, 93, 91, 94, 96, 90, 97, 95, 93, 95, 97, 91, 94, 97.

Solution:

SCORE	TALLY MARKS	f
99	/	1
98		0
97	𝍸	5
96	///	3
95	////	4
94	///	3
93	𝍸 /	6
92	//	2
91	///	3
90	//	2
		N 29

In discussing the data thus arranged, using statistical language, we may say that the score 97 has a frequency of 5, meaning that it occurred that number of times; the score 95 has a frequency of 4. The score 98 has a frequency of zero, because it did not occur at all; but we have to include it in our table, for any frequency distribution, to be correctly presented, must have a place for every possible measure between the lowest and highest of the series. That is, it must be *continuous within the range.* In this case the range is 90–99, so we use 99 for the top step, 90 for the bottom step, and include all the numbers between. This, of course, is done before we begin to tally the scores, and we need not know at the start what numbers will be used excepting these two.

PROBLEMS TO SOLVE

Arrange each series in a frequency distribution:

1. 96, 98, 94, 91, 97, 93, 99, 95, 97, 95, 92, 96, 90, 95, 94, 93, 97, 95, 94, 96, 94, 99, 94, 92, 97.
2. 77, 73, 80, 74, 70, 72, 82, 76, 79, 76, 78, 77, 81, 75, 77, 74, 76, 79, 73, 75, 71, 74, 76, 80, 77, 78, 75, 76, 72, 75, 81, 78, 76, 73, 75, 80, 73.
3. 90, 95, 88, 97, 91, 93, 85, 96, 98, 92, 90, 96, 99, 89, 90, 94, 93, 87, 94, 98, 90, 92, 91, 95, 93, 87, 90, 92, 96, 91, 90, 98, 94, 86, 92.
4. 67, 54, 60, 62, 60, 57, 61, 56, 58, 63, 53, 57, 62, 68, 55, 66, 64, 61, 59, 60, 57, 59, 66, 54, 56, 58, 55, 54, 57, 56, 58, 63, 61, 60, 60, 58, 57, 63, 59, 57, 58.
5. 21, 17, 13, 16, 19, 22, 18, 16, 18, 20, 19, 14, 17, 19, 17, 21, 19, 16, 14, 19, 20, 18, 19, 17, 18, 19, 22, 18, 17, 19, 18, 20, 20, 21, 19, 16, 14, 17.
6. 16, 22, 23, 22, 19, 20, 25, 14, 26, 24, 22, 21, 27, 23, 20, 17, 25, 22, 19, 24, 23, 20, 26, 17, 21, 25, 22, 20, 21, 23, 22, 17, 24, 24, 22, 24.
7. 9, 14, 17, 21, 13, 18, 12, 6, 13, 15, 5, 10, 17, 11, 12, 9, 15, 17, 12, 14, 13, 17, 18, 14, 17, 14, 10, 11, 6, 9, 6, 10, 14, 12, 12, 11, 12, 15, 18, 14, 13, 14, 10, 9, 10.
8. 12, 2, 9, 7, 8, 13, 6, 8, 2, 7, 0, 8, 6, 9, 11, 2, 8, 1, 11, 8, 12, 6, 8, 2, 9, 8, 12, 7, 6, 9, 8, 7, 11, 7, 6, 12, 6, 11, 8, 7, 8, 1, 8, 13, 7, 12, 2, 9, 8, 9, 6, 12.
9. 10, 8, 13, 6, 9, 8, 11, 14, 15, 11, 9, 12, 6, 3, 8, 6, 10, 13, 9, 10, 13, 15, 5, 8, 3, 6, 8, 11, 9, 6, 12, 5, 14, 10, 9, 6, 8, 10, 13, 6, 8, 6, 9, 5, 11, 10, 9, 13, 14, 6, 10, 8.
10. 58, 52, 53, 54, 51, 54, 52, 57, 56, 60, 53, 55, 57, 56, 51, 53, 57, 54, 55, 53, 59, 57, 53, 58, 53, 57, 52, 56, 53, 57, 54, 54, 57, 53, 57, 53, 58, 50, 59, 53, 55, 57, 52, 58, 54.
11. 133, 136, 131, 125, 129, 127, 132, 129, 133, 131, 125, 128, 135, 131, 129, 130, 127, 130, 123, 129, 132, 135, 129, 127, 125, 128, 130, 132, 125, 129, 133, 132, 135, 128, 125, 130, 129, 128, 132, 127, 128, 132, 127, 129, 128, 131, 129, 127, 125, 131, 130, 136.

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13. 91, 94, 87, 98, 85, 93, 89, 97, 91, 96, 86, 92, 90, 96, 95, 91, 99, 89, 86, 72, 83, 90, 88, 94, 91, 97, 88, 95, 90, 95, 93, 89, 94, 91, 94, 91, 99, 92, 98, 84, 91, 95, 89, 94, 88, 91, 95, 92, 88, 91, 93, 92, 94, 89.

14. 76, 74, 82, 72, 73, 78, 81, 81, 85, 81, 78, 75, 72, 71, 85, 82, 70, 77, 84, 81, 77, 73, 77, 71, 69, 76, 83, 76, 77, 67, 85, 84, 76, 80, 71, 72, 74, 75, 71, 72, 76, 83, 85, 83, 76, 77, 75, 67, 67, 72, 80, 69, 83, 78, 85, 69, 77, 78, 65, 73, 76, 67, 74, 76, 78, 67, 75, 70, 83, 78, 81, 80, 76.

15. 147, 139, 134, 150, 141, 149, 135, 132, 145, 140, 136, 147, 140, 149, 139, 143, 138, 142, 148, 141, 136, 135, 145, 134, 140, 135, 149, 140, 136, 143, 136, 143, 148, 141, 140, 145, 138, 143, 138, 142, 140, 145, 141, 139, 142, 141, 140, 136, 142, 139, 140, 147, 138, 134, 140, 142, 139, 143, 141, 147, 141, 140, 137, 143, 136.

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CHAPTER III

INTERVALS AND THEIR VALUES

Explanation. — Each of the groups, or steps, on the measuring scale into which a frequency distribution is divided is called an *interval.* Here is a frequency distribution with ten intervals:

INTERVAL	f
65	2
64	4
63	7
62	3
61	5
60	3
59	1
58	0
57	1
56	1
	N 27

Thus we say the *interval* 63 has a frequency of 7; the *interval* 58 has a frequency of 0; the highest *interval* in the table is 65; the lowest *interval* is 56. The distance between these *intervals* is 1 point, so we say the size of the *interval* is 1.

In statistical work it is often advisable to condense frequency distributions for convenience in handling. Should we wish to do so with this table, we might put the same frequencies in a distribution having an interval of 2, thus:

INTERVAL	f
64–65	6
62–63	10
60–61	8
58–59	1
56–57	2
	N 27

Here the interval 62–63 (read "62 to 63") has a frequency of 10. The lowest interval is 56–57, with a frequency of 2; the highest interval is 64–65, with a frequency of 6. The value of N remains the same, because the same cases were used for both tables.

We may use intervals of any size, depending on the kind of data with which we deal. The purpose of using intervals is to enable us to put the numbers in the most usable form and to save the time which would be required in handling many individual numbers.

Suppose we wish to write a series of intervals, beginning with 92–93, and going downward. We would write:

92–93
90–91
88–89
86–87
84–85
82–83
80–81, etc.

We could distribute numbers in these intervals by putting each number in its proper place; thus a score of 84 would be tallied in the interval 84–85; a score of 89 would go in the interval 88–89, etc.

ILLUSTRATIVE PROBLEM. — Distribute the following scores in intervals of 2, beginning (at the top) with the interval 98–99:

95, 98, 92, 94, 88, 95, 96, 91, 94, 93, 98, 95, 92, 89, 93, 97, 92, 97.

Solution:

INTERVAL	TALLY MARKS	f
98–99	//	2
96–97	///	3
94–95	~~////~~	5
92–93	~~////~~	5
90–91	/	1
88–89	//	2
		N 18

ILLUSTRATIVE PROBLEM. — Write a series of ten intervals beginning with 95–99.

Solution:

95–99
90–94
85–89
80–84
75–79
70–74
65–69
60–64
55–59
50–54

ILLUSTRATIVE PROBLEM. — Write a series of intervals beginning with 90–99.

Solution:

90–99
80–89
70–79
60–69
50–59
40–49
30–39
20–29
10–19
0–9

ILLUSTRATIVE PROBLEM. — Write a series of intervals beginning with 80–99. *Solution:*

80–99
60–79
40–59
20–39
0–19

PROBLEMS TO SOLVE

Write a series of ten intervals beginning with each of the following as the top interval:

1.	100–109	**6.**	80–83	**11.**	310–319	**16.**	75–79
2.	55–59	**7.**	120–129	**12.**	122–123	**17.**	90–95
3.	40–41	**8.**	450–474	**13.**	100–103	**18.**	250–259
4.	39–41	**9.**	25	**14.**	130–139	**19.**	63–65
5.	180–199	**10.**	105–109	**15.**	112–113	**20.**	72–75

21. Arrange the following scores in a frequency distribution having 90–99 for the top interval:

68, 92, 70, 45, 63, 72, 86, 58, 37, 55, 73, 44, 62, 81, 66, 59, 64, 72, 53, 68, 75, 67, 56.

NOTE. — First write the intervals, then tally the scores. Write only as many intervals as will be needed to include the range.

22. Distribute similarly in intervals beginning with 75–79:
52, 63, 72, 43, 38, 61, 56, 72, 63, 52, 56, 78, 46, 56, 51, 67, 48, 55, 52, 68, 51, 73, 52, 43, 69, 44, 50, 78, 58, 46, 54, 70, 59, 63.

23. Distribute in intervals beginning with 80–99:
92, 57, 28, 43, 56, 41, 75, 38, 92, 15, 50, 60, 25, 66, 45, 12, 38, 96, 22, 61, 25, 50, 18, 36, 62, 40, 28, 56, 43, 75, 58, 48.

24. Distribute in intervals beginning with 62–63:
53, 46, 44, 60, 57, 54, 53, 60, 59, 46, 50, 57, 62, 56, 53, 54, 58, 57, 52, 50, 60, 52, 54, 55, 63, 51, 54, 53, 46, 59, 50, 57, 60, 46, 54, 53, 50, 58, 59, 53, 57, 55, 51, 46, 54, 53, 60, 52, 61.

25. Distribute in intervals beginning with 33–35:
25, 16, 21, 31, 18, 22, 13, 28, 21, 30, 19, 31, 25, 34, 15, 29, 21, 27, 22, 19, 28, 34, 16, 22, 28, 30, 14, 16, 20, 19, 13, 23, 25, 27, 30, 20, 24, 21, 20.

26. Distribute in intervals of 2: (Top interval 32–33)
27, 33, 30, 18, 9, 22, 17, 27, 31, 20, 15, 13, 11, 15, 19, 23, 25, 29, 13, 16, 18, 8, 14, 22, 27, 12, 13, 23, 25, 20, 16, 19, 13, 29, 31, 20, 24, 20, 17, 21, 24, 30, 31, 13, 10, 29.

27. Distribute in intervals of 3: (Top interval 99–101)
67, 93, 68, 62, 73, 99, 60, 100, 66, 75, 76, 84, 88, 76, 91, 93, 68, 72, 71, 79, 88, 80, 93, 95, 61, 75, 77, 83, 82, 86, 67, 65, 74, 82, 97, 80, 81, 73, 72, 85, 80, 70, 69, 62, 70, 71, 73.

28. Distribute in intervals of 5: (Top interval 90–94)
93, 90, 87, 64, 63, 40, 58, 87, 86, 75, 70, 71, 62, 53, 58, 69, 68, 90, 87, 61, 43, 42, 42, 50, 55, 77, 70, 60, 63, 68, 74, 90, 87, 80, 53, 61, 66, 68, 74, 90, 87, 80, 53, 61, 66, 68, 64, 60, 69, 70, 59, 56, 57, 40.

29. Distribute in intervals of 5: (Top interval 150–154)
125, 133, 98, 120, 154, 110, 85, 88, 122, 125, 138, 126, 119, 114, 118, 128, 85, 97, 100, 105, 150, 144, 88, 121, 113, 100, 99, 111, 120, 125, 124, 122, 127, 140, 110, 123, 125, 134, 141.

30. Distribute in intervals of 10: (Top interval 140–149)
134, 127, 90, 62, 143, 82, 89, 96, 100, 99, 84, 73, 82, 80, 96, 75, 70, 68, 93, 90, 85, 92, 69, 73, 130, 138, 77, 84, 96, 90, 100, 115, 67, 73, 70, 94, 101, 113, 93, 90, 87, 86, 84.

31. Distribute in intervals of 15: (Top interval 90–104)
36, 62, 83, 72, 73, 100, 46, 53, 13, 42, 73, 0, 37, 61, 77, 95, 103, 46, 42, 32, 67, 50, 48, 78, 50, 52, 73, 51, 72, 88, 80, 30, 46, 19, 12, 44, 63, 7, 38, 40, 68, 57, 56, 60, 100, 61, 77, 88, 70, 81.

32. Distribute in intervals of 20: (Top interval 80–99)
92, 63, 84, 96, 14, 36, 48, 87, 32, 93, 82, 70, 56, 58, 62, 87, 62, 70, 60, 42, 37, 93, 80, 82, 76, 58, 66, 54, 41, 80, 97.

33. Distribute in intervals of 15: (Top interval 90–104)
87, 63, 54, 18, 100, 13, 36, 48, 16, 43, 76, 19, 32, 73, 91, 40, 60, 50, 16, 82, 56, 99, 55, 88, 0, 35, 61, 15, 47, 52, 65, 48, 47, 31, 68, 92, 18, 32, 49, 63, 72, 19, 0, 13, 45, 63, 68, 75, 52, 49, 37, 58, 63, 72, 38, 45, 75, 60.

How to decide what the top interval will be. — In each of the preceding problems you have been told what the top intervals should be. Here are some problems in which you are not told. The top interval may best be found by locating the highest measure in the series, and then deciding on an interval which *begins with a multiple of the size of the interval*, and which will include the highest measure.

ILLUSTRATIVE PROBLEM. —

Distribute in intervals of 5:
83, 75, 92, 96, 84, 73,
90, 81, 85, 76, 65, 68,
88, 69, 72, 91, 70, 83,
82, 74, 68, 75, 77, 86.

Solution:
The highest number in the series is 96; hence we will choose 95–99 as the top interval. The lowest interval of this distribution will be 65–69.

ILLUSTRATIVE PROBLEM. —

Distribute in intervals of 3:
64, 68, 54, 57, 73, 71, 74,
70, 63, 68, 62, 58, 61, 72,
80, 75, 64, 77, 75, 68, 73,
79, 80, 65, 72, 80, 78, 71.

Solution:
The highest number in the series is 80; hence the top interval will be 78–80. The lowest interval will be 54–56.

ILLUSTRATIVE PROBLEM. —

Distribute in intervals of 10:
70, 78, 83, 62, 55, 73, 89,
76, 64, 23, 29, 46, 50, 58,
88, 86, 18, 24, 53, 68, 72,
75, 93, 89, 66, 93, 85, 52.

Solution:
The top interval will be 90–99, to accommodate the highest score, 93. The bottom interval will be 10–19, since the lowest score is 18.

PROBLEMS TO SOLVE *(Continued)*

34. Distribute in intervals of 10:
54, 58, 62, 65, 50, 32, 86, 78, 63, 60, 54, 48, 35, 39, 30, 43, 28, 62, 47, 58, 35, 69, 72, 75, 45, 56, 55, 81, 45, 58, 40, 29, 46, 55, 68, 70.

35. Distribute in intervals of 5:
42, 56, 28, 37, 16, 12, 24, 48, 29, 38, 46, 53, 62, 37, 28, 35, 49, 27, 48, 36, 37, 26, 42, 18, 53, 31, 58, 24, 56, 25, 43, 25, 38, 19, 41, 52, 36, 38, 32, 48, 56, 27, 45, 39, 24, 56, 37, 29, 47, 38.

36. Distribute in intervals of 2:
54, 56, 41, 37, 53, 58, 65, 43, 59, 46, 39, 36, 34, 58, 46, 42, 61, 53, 44, 54, 60, 39, 47, 55, 58, 50, 40, 55, 43, 62, 42, 35, 60, 50, 47, 48, 41, 49, 40, 51, 57, 46, 40, 62, 48, 40, 50, 53, 40, 35, 49, 56.

37. Distribute in intervals of 5:
142, 115, 153, 138, 162, 127, 148, 156, 167, 159, 113, 136, 150, 149, 142, 152, 137, 132, 146, 160, 150, 138, 127, 116, 132, 135, 154, 138, 139, 144, 118, 126, 123, 142, 119, 132, 158, 147, 125, 120, 138, 162, 120, 138, 142, 157, 160, 135, 140, 115, 129, 142, 153, 168, 157, 163, 152, 131, 142, 129, 127, 133.

38. Distribute in intervals of 15:
73, 92, 36, 95, 48, 32, 13, 68, 35, 10, 45, 93, 78, 62, 33, 18, 47, 39, 76, 46, 31, 92, 73, 58, 35, 95, 36, 102, 33, 16, 107, 64, 38, 110, 49, 37, 70, 60, 113, 38, 18, 66, 49, 62, 70, 43, 79, 46, 48, 93, 39, 19, 68, 47, 63, 105, 48, 78, 50, 45, 38, 79, 62, 47, 65, 35, 72, 43, 71, 95, 100, 40, 102, 33, 6, 14.

CHAPTER IV

INTERVAL SIZES

Explanation. — The size of an interval is the number of individual measures it includes within its entire span. Usually we write only the first and last number; thus 35–39 is an interval of 5, representing the measures 35, 36, 37, 38 and 39. Similarly, 8–9 is an interval of 2; 27–29 is an interval of 3; 90–99 is an interval of 10; 80–99 is an interval of 20; 72–77 is an interval of 6.

ILLUSTRATIVE PROBLEMS, *with solutions.* — Indicate the size of each of the following intervals:

INTERVAL	SIZE	INTERVAL	SIZE
(*a*) 60–69	10	(*f*) 65–69	5
(*b*) 50–54	5	(*g*) 52–55	4
(*c*) 78–79	2	(*h*) 100–119	20
(*d*) 39–41	3	(*i*) 90–91	2
(*e*) 75–99	25	(*j*) 75*	1

PROBLEMS TO SOLVE

Indicate the size of each of the following intervals:

1. 80–89	**8.** 5–9	**15.** 10–19	**22.** 100–109
2. 60–64	**9.** 50–59	**16.** 42–44	**23.** 0–2
3. 80–99	**10.** 0–4	**17.** 60–69	**24.** 20–24
4. 6–7	**11.** 18–19	**18.** 15–19	**25.** 98–99
5. 20–29	**12.** 70–79	**19.** 21–27	**26.** 40–49
6. 125–149	**13.** 99–101	**20.** 30–39	**27.** 50–99
7. 25–29	**14.** 10–14	**21.** 48–50	**28.** 60–74

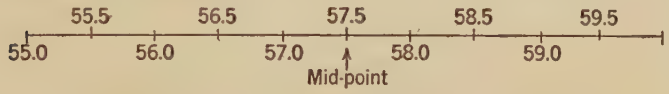
* Any single number is an interval of 1. This interval, 75, could be written 75.0–75.9, but it is not necessary to do so.

29. 55–59
30. 0–9
31. 110–114
32. 45–59
33. 90–95
34. 90–91
35. 0–6
36. 92–93
37. 450–459
38. 35–39
39. 290–299
40. 140–159
41. 30–44
42. 96–98
43. 76–77
44. 0–7
45. 93–95
46. 320–329
47. 8–11
48. 180–182
49. 20–23
50. 81–83
51. 30–34
52. 0–5
53. 82–83
54. 160–169
55. 50–54
56. 0–3
57. 96–97
58. 78–79
59. 18–23
60. 120–124
61. 0–8
62. 80–81
63. 130–139
64. 0–5
65. 94–95
66. 15–29
67. 115–119
68. 0–14
69. 56–59
70. 160–179
71. 12–15
72. 90–92
73. 50–99
74. 88–89
75. 84–86
76. 84–85
77. 190–199
78. 40–44
79. 6–11
80. 270–279
81. 86–87
82. 45–49
83. 0–4
84. 75–89
85. 125–129
86. 150–159
87. 30–35
88. 460–469
89. 75–99
90. 360–369
91. 32–35
92. 105–109
93. 14–20
94. 87–89

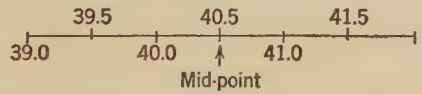
CHAPTER V

THE INTERVAL MID-POINT

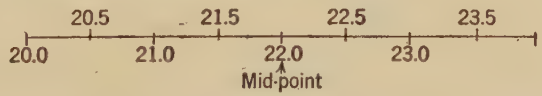
Explanation. — Every interval has a mid-point, an important measure of its value, which is the point midway between its two extremes. Thus the interval 55–59 has for its mid-point 57.5. This may be illustrated as follows:



Likewise, the mid-point of interval 39–41 is 40.5:



The mid-point of the interval 20–23 is 22.0:



Illustrative problems, *with solutions.* — Indicate the mid-point of each of the following intervals:

INTERVAL	MID-POINT
50–59	55.0
90–99	95.0
85–89	87.5
0–4	2.5
75–89	82.5
0–9	5.0
6–7	7.0
83	83.5

Note. — The mid-point of any interval may be found by adding one-half the *size of the interval* to its beginning value.

PROBLEMS TO SOLVE

Indicate the mid-point of each of the following intervals:

1.	90–99	**21.**	5–9	**41.**	10–19
2.	45–49	**22.**	30–31	**42.**	98
3.	8–9	**23.**	70–79	**43.**	35–39
4.	55–59	**24.**	12–13	**44.**	16–17
5.	80–99	**25.**	60–64	**45.**	100–109
6.	50–59	**26.**	28–29	**46.**	26–27
7.	10–11	**27.**	80–89	**47.**	100–119
8.	60–79	**28.**	40–59	**48.**	75–79
9.	60–69	**29.**	40–44	**49.**	20–21
10.	18–19	**30.**	110–119	**50.**	0–4
11.	50–54	**31.**	14–15	**51.**	140–149
12.	9–11	**32.**	25–29	**52.**	93–95
13.	40–49	**33.**	66–68	**53.**	30–34
14.	160–179	**34.**	20–29	**54.**	120–139
15.	16–19	**35.**	34–35	**55.**	250–259
16.	30–39	**36.**	140–159	**56.**	36–37
17.	60–74	**37.**	0–9	**57.**	0–2
18.	24–27	**38.**	36–47	**58.**	24–25
19.	10–19	**39.**	22–23	**59.**	50–99
20.	32–33	**40.**	56–63	**60.**	21–27

CHAPTER VI

THE MID-INTERVAL

Explanation. — The mid-interval of a frequency distribution is the *interval containing the middle case.* In the following table we may count down from the top to find the middle case (which, of 23 cases, will be the twelfth), and find that it occurs in the interval 85–89:

INT.	*f*	
95–99	3	
90–94	4	
85–89	8	(mid-interval)
80–84	5	
75–79	2	
70–74	1	
	N 23	

ILLUSTRATIVE PROBLEM. —
Find the mid-interval: *Solution:*

INT.	*f*
120–129	3
110–119	5
100–109	9
90–99	12
80–89	7
70–79	5
60–69	0
50–59	2
N	43

INT.	*f*	
120–129	3	
110–119	5	
100–109	9	
90–99	12	(mid-interval)
80–89	7	
70–79	5	
60–69	0	
50–59	2	
N	43	

ILLUSTRATIVE PROBLEM. —
Find the mid-interval: *Solution:*

INT.	f
180–199	1
160–179	3
140–159	9
120–139	13
100–119	10
80–99	5
60–79	4
40–59	2
20–39	2
N	49

INT.	f	
180–199	1	
160–179	3	
140–159	9	
120–139	13	(mid-interval)
100–119	10	
80–99	5	
60–79	4	
40–59	2	
20–39	2	
N	49	

ILLUSTRATIVE PROBLEM. —
Find the mid-interval: *Solution:*

INT.	f
58–59	1
56–57	0
54–55	2
52–53	4
50–51	5
48–49	7
46–47	9
44–45	2
42–43	0
40–41	2
N	32

INT.	f	
58–59	1	
56–57	0	
54–55	2	
52–53	4	
50–51	5	
48–49	7	(mid-interval)
46–47	9	
44–45	2	
42–43	0	
40–41	2	
N	32	

PROBLEMS TO SOLVE

Find the mid-interval of each of the following distributions:

1.

INT.	f
90–99	3
80–89	6
70–79	5
60–69	12
50–59	5
40–49	7
30–39	0
20–29	2
N	40

2.

INT.	f
95–99	1
90–94	2
85–89	3
80–84	10
75–79	11
70–74	10
65–69	5
60–64	1
N	43

3.

INT.	f
155–159	1
150–154	7
145–149	15
140–144	11
135–139	11
130–134	6
125–129	6
120–124	5
115–119	2
110–114	1
N	65

4.

INT.	f
32–33	1
30–31	6
28–29	8
26–27	9
24–25	12
22–23	15
20–21	16
18–19	0
16–17	13
14–15	10

5.

INT.	f
180–199	4
160–179	0
140–159	5
120–139	7
100–119	12
80–99	12
60–79	9
40–59	0
20–39	2
0–19	1

6.

INT.	f
98	3
97	5
96	8
95	9
94	9
93	7
92	9
91	8
90	4
89	4

7.

INT.	f
55–59	4
50–54	5
45–49	0
40–44	8
35–39	15
30–34	24
25–29	18
20–24	13
15–19	12
10–14	7

8.

INT.	f
80–81	5
78–79	3
76–77	8
74–75	10
72–73	12
70–71	10
68–69	12
66–67	7
64–65	4
62–63	3

9.

INT.	f
75	2
74	0
73	0
72	2
71	4
70	3
69	5
68	7
67	12
66	10
65	6

10.

INT.	f
90–99	2
80–89	4
70–79	8
60–69	3
50–59	0
40–49	0
30–39	2
20–29	1
10–19	2

11.

INT.	f
60–62	9
57–59	12
54–56	7
51–53	10
48–50	15
45–47	8
42–44	0
39–41	9
36–38	7
33–35	2

12.

INT.	f
75–79	3
70–74	8
65–69	7
60–64	14
55–59	12
50–54	7
45–49	3
40–44	18
35–39	12

13.

INT.	f
18	3
17	4
16	5
15	8
14	9
13	7
12	8
11	5

14.

INT.	f
90–99	3
80–89	7
70–79	18
60–69	7
50–59	0
40–49	0
30–39	4
20–29	2

15.

INT.	f
80–99	18
60–79	24
40–59	28
20–39	57
0–19	39

Complete the intervals for each of the following frequency distributions and find the mid-interval:

16.

INT.	*f*
80–89	3
	4
	8
	14
	13
	10
	9
	2

17.

INT.	*f*
18–19	1
	1
	4
	5
	8
	12
	2
	2

18.

INT.	*f*
180–199	2
	8
	15
	14
	4
	3
	0
	1

19.

INT.	*f*
120–129	4
	2
	8
	12
	15
	24
	20
	13
	4
	2
	1

20.

INT.	*f*
90–94	3
	8
	12
	10
	8
	7
	2
	4
	1

21.

INT.	*f*
75–79	2
	0
	4
	6
	12
	9
	3
	2
	1
	0
	1

22.

INT.	*f*
110–119	2
	3
	5
	8
	12
	22
	25
	18
	10
	5

23.

INT.	*f*
65–69	6
	8
	11
	10
	42
	30
	16
	11
	4
	2

24.

INT.	*f*
88–89	4
	5
	6
	10
	22
	23
	15
	12
	8
	6

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25.

INT.	f
100–109	5
	5
	10
	10
	22
	25
	10
	8
	5
	2

26.

INT.	f
200–209	6
	6
	12
	18
	20
	21
	29
	17
	13
	0
	2
	4
	2

27.

INT.	f
67	3
	5
	15
	15
	25
	10
	10
	7
	6
	5

28.

INT.	f
140–149	5
	6
	8
	15
	25
	10
	30
	10
	15
	14
	4
	3
	0
	0
	2

29.

INT.	f
300–302	1
	3
	4
	9
	15
	21
	25
	20
	11
	10
	5
	0
	2
	1

30.

INT.	f
420–439	3
	0
	0
	7
	18
	26
	22
	10
	0
	7
	1

CHAPTER VII

THE CHOICE OF INTERVALS

Explanation. — Suppose we have a set of untabulated scores which we wish to place in a frequency distribution. We must choose a suitable size of interval, and write down the intervals in their correct order before we can distribute the scores.

The chief purpose of using intervals is to permit arrangement of the data in convenient, workable form. It is not practicable to deal with large numbers of individual scores, because of the amount of time required and the increased chance of error. So we use intervals to condense the data, having care not to destroy the significance of the measures. There is no single rule regarding the extent to which it is safe to condense a distribution, but it is common statistical procedure to make tables of about ten to twenty intervals. To use more than twenty usually spreads the data over too much space, and makes too many separate steps to take in computations. On the other hand, a very small distribution, containing fewer than about ten intervals, may be so condensed as to cause the data to lose its significance for many of the types of measures which we obtain from the use of intervals. There will be occasions requiring the use of unusually small or unusually large distributions; but ordinarily if we plan to have from ten to twenty intervals we will be following good procedure.

The decision as to what size of interval to use de-

pends first upon the range of the measures. Suppose, for example, we have a series of scores ranging from 50 to 169. This is a range of 120 points. If we should attempt to place the scores in intervals of one, we should have 120 intervals — too large a tabulation for convenience. If we place them in twos, there would be 60 intervals; by fours, there would be 30; by fives, 24; by tens, 12; by twenties, 6. Following the suggestions just advanced, we would probably choose the interval ten, for that would give us a distribution of twelve intervals.

There is another factor to consider in choosing the size of interval. Some sizes are easier to work with than others. Probably the most convenient size is ten; then perhaps five; while fifteen, three, seven, twenty, and many others are awkward to handle. However, the main purpose should be kept in mind; to get the measures into convenient and condensed (but not too condensed) form. If at any time it should appear that any one of several intervals might yield a distribution of suitable size, choose the size with which you find you can most conveniently work. Intervals of five and ten are by far the most often used.

ILLUSTRATIVE PROBLEM. — What would be a suitable size of interval to use in distributing a series of scores ranging from 40 to 176, and what would be the top interval?

Solution: The range is approximately 140 points. Intervals of ten would be suitable. The top interval would be 170–179.

ILLUSTRATIVE PROBLEM. — What size of interval would you use for a series of scores ranging from 50 to 74?

Solution: The range is 25 points. Intervals of two, beginning with 74–75, would be appropriate.

ILLUSTRATIVE PROBLEMS. — Suggest an appropriate size and indicate the top interval for distributing each of the following ranges:

(*a*) 15–142; (*b*) 25–99; (*c*) 10–40; (*d*) 100–396; (*e*) 15–32.

Solution:

PROBLEM	SIZE OF INTERVAL	TOP INTERVAL
(*a*)	10	140–149
(*b*)	5	95–99
(*c*)	2	40–41
(*d*)	20	380–399
(*e*)	1	32

ILLUSTRATIVE PROBLEM. — Arrange the following scores in a frequency distribution, using intervals of appropriate size:

73, 82, 48, 93, 76, 51, 65, 86, 57, 77, 94, 52, 97, 73, 54, 66, 48, 80, 70, 95, 66, 56, 68, 62, 43, 67, 75, 62, 83, 52, 60, 91, 78, 77, 93, 46.

Solution:

INT.	TALLY MARKS	*f*
95–99	//	2
90–94	////	4
85–89	/	1
80–84	///	3
75–79	~~////~~	5
70–74	///	3
65–69	~~////~~	5
60–64	///	3
55–59	//	2
50–54	////	4
45–49	///	3
40–44	/	1
		N 36

PROBLEMS TO SOLVE

Arrange each set of scores in a frequency distribution having intervals of appropriate size:

1. 124, 163, 102, 183, 94, 135, 72, 143, 104, 165, 130, 85, 175, 143, 115, 62, 105, 150, 122, 168, 175, 114, 80, 120, 145, 96, 135, 82, 128, 166, 137, 75, 125, 182, 163, 139, 105, 132.
2. 42, 83, 47, 26, 88, 57, 70, 54, 93, 62, 30, 66, 44, 27, 33, 60, 85, 67, 90, 45, 73, 58, 30, 74, 88, 96, 58, 70, 42, 65, 43, 66, 97, 70, 92, 51, 66, 32, 65, 93, 60, 42, 67, 44, 83.
3. 28, 56, 72, 45, 30, 50, 67, 43, 48, 36, 18, 42, 23, 70, 52, 65, 54, 78, 47, 66, 35, 52, 38, 57, 78, 35, 22, 43, 56, 52, 33, 37, 54, 47, 40, 47, 70, 42, 38, 55, 50, 71, 58, 36, 47, 26, 30, 73, 56, 42, 37, 45.
4. 86, 123, 54, 75, 62, 115, 58, 57, 64, 25, 73, 116, 68, 75, 124, 63, 115, 58, 72, 25, 70, 118, 86, 27, 66, 83, 67, 118, 84, 22, 63, 85, 52, 23, 56, 74, 58, 65, 73.
5. 40, 45, 35, 43, 37, 46, 40, 34, 41, 38, 44, 48, 43, 37, 32, 39, 41, 35, 40, 35, 46, 41, 35, 39, 45, 38, 48, 37, 40, 34, 39, 44, 41, 37, 41, 35, 41, 34, 44, 48, 39, 32, 38, 44, 40, 37, 45, 34, 40, 46, 42, 32, 43.
6. 35, 82, 120, 85, 45, 93, 33, 105, 62, 136, 50, 18, 92, 75, 122, 62, 100, 42, 90, 33, 82, 25, 84, 104, 53, 112, 65, 120, 73, 20, 75, 33, 115, 68, 104, 38, 72, 146, 75, 95, 43, 28, 60, 109, 137, 94, 43, 86, 67, 98, 59, 76, 38, 56, 27, 115, 79, 44, 70, 112, 85, 48, 65, 105, 97, 76.
7. 53, 42, 56, 67, 46, 55, 53, 46, 38, 50, 64, 54, 49, 53, 41, 48, 64, 50, 41, 47, 52, 66, 57, 46, 53, 49, 40, 48, 61, 51, 40, 49, 57, 50, 43, 55, 64, 55, 47, 40, 38, 38, 53, 60, 55, 50, 53, 46, 48.
8. 78, 63, 75, 46, 63, 76, 63, 72, 18, 44, 50, 30, 42, 64, 32, 52, 60, 33, 50, 73, 34, 46, 23, 17, 46, 16, 66, 42, 50, 20, 46, 77, 53, 22, 45, 62, 33, 54, 33, 45, 70, 54, 62, 41, 54, 32, 50, 72, 40, 17, 46, 73.

9. 35, 49, 40, 60, 42, 25, 42, 34, 60, 26, 41, 50, 35, 45, 33, 48, 43, 62, 25, 44, 48, 25, 40, 45, 61, 44, 48, 40, 48, 35, 60, 35, 50, 34, 43, 45, 24, 39, 45, 24, 41, 60, 44, 35, 42, 43, 44, 34, 60, 45, 26, 60.

10. 48, 107, 30, 136, 25, 95, 45, 110, 75, 130, 70, 15, 122, 162, 78, 80, 42, 95, 70, 160, 100, 25, 62, 105, 35, 140, 90, 45, 90, 30, 120, 60, 13, 65, 118, 80, 136, 115, 38, 75, 80, 114, 140, 35, 78, 62, 123, 47, 80, 15, 152, 65, 92, 137, 100, 80, 52.

Suggest a suitable top interval for a frequency distribution for each of the following ranges:

11.	0–75	**19.**	0–13	**27.**	0–227	**35.**	105–256
12.	8–140	**20.**	100–248	**28.**	0–150	**36.**	7–85
13.	5–50	**21.**	10–170	**29.**	10–84	**37.**	0–50
14.	5–79	**22.**	1–45	**30.**	1–30	**38.**	3–24
15.	12–30	**23.**	40–85	**31.**	8–175	**39.**	6–83
16.	0–140	**24.**	50–195	**32.**	15–32	**40.**	0–200
17.	15–320	**25.**	0–16	**33.**	5–80	**41.**	5–100
18.	6–176	**26.**	24–378	**34.**	100–144	**42.**	75–83

PART II

MEASURES OF CENTRAL TENDENCY

CHAPTER VIII

THE MEANING OF CENTRAL TENDENCY

Definition. — A central tendency is a measure which is used to represent a group of measures. It can perhaps best be illustrated by the common average, a widely known measure. When we average a group of numbers we find a single measure, usually about half-way through the range, which we use to *represent* the entire group. Thus if the ages of a small class of pupils are 10, 8, 12, 7, 9, 10, 8, 9, 11, 12, their average is 9.6, which is about half-way through the range, 7–12. We may find it convenient to use 9.6 as an index of the age of these pupils, not that all the pupils are that age, but because that is the age they would be if their combined ages were distributed equally among them. If another class has an average of 12.3, we may say that its pupils are 2.7 years older than the first group, meaning that there is 2.7 years difference between the two central tendencies. It is a convenience to be able to use a single number, such as 9.6 or 12.3, to tell about a group, instead of having to use the entire series of measures.

Practical uses of central tendency. — When the census reports give data concerning the size of families, the taxable wealth of the population, the value of farm land, etc., the comparisons are based on central tendencies of groups, often without reference at all to individual cases. Similarly, a teacher frequently reports a single measure representing the standing of a

class of pupils. A school report on average daily attendance is another illustration of how a central tendency may represent a group. Without the use of central tendencies we would be at a loss for making practical deductions from measured facts; for it is not possible, in comparing one group with another, to separately compare all the individual cases.

One of the important things to remember about a central tendency is that while it may appropriately *represent* the group of measures from which it is calculated, it does so only in a general way. It shows the *tendency* of the group according to its *central* features. Individual cases in the group may be greatly different from the measure which represents them. An average annual rainfall of 18 inches for a twenty year period may include several dry years having rainfall of less than three inches; or it may include several flood years having rainfall greatly exceeding the average. A class of pupils having an average test score of 85 might contain a pupil scoring as low as 20, or even zero. So we should think of central tendencies as representing, but not fully indicating, their respective groups.

Inasmuch as the central tendency is often used for drawing general conclusions, care should be taken to see that the group is large enough to warrant generalization. The average height of ten-year-old boys, for example, could not correctly be determined from the average of twenty or thirty cases, but if a thousand boys were measured, we might be justified in using their average for such a purpose. The problems given in this book are mostly small, and are intended to supply practice material in method, rather than to indicate what sizes of groups will be needed for various purposes.

Types of central tendency. — Three general types of central tendency are shown in this book: (1) Averages based on a sort of "leveling" process, as if the sum of the values were divided equally among the members of the group. The common average, or arithmetic mean, represents this type. (2) Central tendencies obtained by using the middle measure of the group, as in the mid-score and median. (3) The point of greatest frequency in the group: the mode or modal interval.

These measures are computed in quite different ways, but they are often the same, or similar. In each case the measure will be found at or near the center of the group. In some groups, the mean, the median, and the mode occur at exactly the same point. There is usually a slight difference, however, which will vary with the size and nature of the group being studied. In general, it may be said that the significance of any central tendency depends upon the size and nature of the group.

The central tendencies referred to in the succeeding chapters may be outlined as follows:

OBTAINED FROM A SIMPLE SERIES	OBTAINED FROM A FREQUENCY DISTRIBUTION	METHOD OF OBTAINING
Arithmetic mean (Common average)	Arithmetic mean	By averaging, or evenly dividing, the measures
Mid-score	Median	The middle measure
Mode	Modal interval	The most frequent measure

REFERENCES ON CENTRAL TENDENCY

Chaddock, *Principles and Methods of Statistics*, ch. VI–VIII.
Garrett, *Statistics in Psychology and Education*, ch. II.
Hines, *A Guide to Educational Measurements*, ch. II–III.
Holzinger, *Statistical Methods for Students in Education*, ch. VI.
Kelley, *Statistical Method*, ch. III.
King, *Elements of Statistical Method*, ch. XII.
Lincoln, *Beginnings in Educational Measurement*, ch. III.
McCall, *How to Measure in Education*, ch. XV.
Odell, *Educational Statistics*, ch. III.
Otis, *Statistical Method in Educational Measurement*, ch. II.
Rugg, *A Primer of Graphics and Statistics*, ch. III.
Thurstone, *The Fundamentals of Statistics*, ch. X–XII.
Trabue, *Measuring Results in Education*, ch. IX.

CHAPTER IX

THE COMMON AVERAGE, OR ARITHMETIC MEAN

Explanation. — Perhaps the best-known central tendency is the *common average*, or *mean*.* It has the advantage of being simple to calculate, and is so widely used that it is not usually thought of as being one of the technical measures of statistics. Its simplicity makes it the most easily understood of all the measures of central tendency. If five boys weigh 65, 84, 96, 92, and 58 pounds, respectively, we find their *average* weight to be 79 pounds. This means that if the total weight should be equally divided among the five boys, they would each weigh 79 pounds.

The common average is found by *dividing the sum of the measures by the number of cases.* In the foregoing illustration,

65
84
96 } (the measures)
92
58

(the number of cases) 5)395 (the sum of the measures)
79 (the common average)

This procedure is represented for statistical purposes by the formula:

$$M = \frac{\Sigma m}{N}$$

in which M stands for the *Mean* (used statistically for *average*); Σ (capital S of Greek alphabet) for " the sum of "; m stands for *measures;* N for " number of cases ".

*This average is sometimes referred to as the *arithmetic mean,* to distinguish it from other types of average. It is further dealt with in chap. 10.

The formula, it will be observed, is merely a way of representing the method by which we solve the problem. Referring again to the weights of the five boys, we may show our procedure thus:

(Formula) $$M = \frac{\Sigma m}{N}$$

$$M = \frac{395}{5} = 79 \text{ (Ans.)}$$

In obtaining the common average it is not necessary to arrange the numbers in order, inasmuch as we need only to find their sum.

The common average, like other central tendencies, will usually approximate the center of the range. In the foregoing problem the range is 58–96, the center of which is 77. The actual average is 79. The tendency to this approximation is more marked when the number of cases is large.

ILLUSTRATIVE PROBLEMS, *with solutions.* — Find the common average of each column (answers to one decimal place):

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
83	48	9	384	96
96	47	8	456	99
72	50	7	247	96
54	65	6	153	96
65	53	9	200	98
5)370	82	0	300	97
74.0	85	8	304	99
(Ans.)	7)430	6	240	94
	61.4	8)53	350	90
	(Ans.)	6.6	400	98
		(Ans.)	10)3034	95
			303.4	90
			(Ans.)	12)1148
				95.6
				(Ans.)

PROBLEMS TO SOLVE

Find the common average (answers to one decimal place):

1. 75, 83, 70, 69, 58, 73, 75, 92, 89, 70.
2. 40, 38, 27, 31, 36, 45, 46, 73, 70, 36.
3. 130, 142, 100, 98, 97, 100, 105, 160, 132, 99, 97, 90.
4. 10, 18, 9, 7, 8, 14, 13, 10, 13, 10, 9, 5, 8, 7, 13, 15.
5. 36, 38, 40, 35, 37, 38, 30, 32, 35, 39, 37, 30, 36, 32.
6. 97, 90, 96, 98, 99, 70, 99, 75, 80, 83, 80, 92, 95, 86, 83.
7. 84, 63, 67, 68, 70, 72, 83, 80, 96, 99, 72, 70, 63, 68, 69, 70, 75, 64, 52, 58.
8. 39, 30, 36, 27, 38, 42, 45, 38, 39, 46, 37, 32, 36, 45, 40, 30, 42, 51, 42, 50, 68, 37, 28, 39.
9. 56, 43, 49, 42, 85, 67, 80, 73, 64, 53, 48, 57, 60, 50, 40, 35, 45, 30, 27, 82, 63, 82, 60, 75, 40.
10. 3, 8, 7, 4, 6, 5, 2, 3, 8, 5, 4, 4, 3, 9, 4, 2, 2, 3, 1, 6, 5, 4, 8, 3, 7, 7, 7, 6, 4, 3.
11. 152, 146, 185, 161, 160, 139, 146, 157, 160, 130, 163, 180, 190, 128, 150, 165, 138, 100, 170, 140, 153, 129.
12. 95, 90, 87, 93, 90, 85, 80, 82, 86, 93, 90, 97, 96, 95, 80, 82, 85, 88, 90, 88, 75, 83, 78, 80, 87.
13. 50, 52, 56, 50, 58, 60, 63, 64, 63, 65, 70, 68, 73, 75, 63, 82, 77, 66, 50, 65, 75, 60, 70, 80, 81, 70, 75, 76, 82.
14. 92, 98, 95, 96, 99, 80, 86, 88, 84, 82, 89, 90, 85, 80, 81, 86, 87, 80, 75, 80, 83, 80, 76, 79, 82, 80.
15. 90, 93, 98, 96, 97, 97, 97, 99, 99, 99, 100, 100, 100, 100, 100, 101, 102, 102, 104, 105, 105, 110, 110, 123.
16. 85, 92, 96, 88, 83, 90, 97, 99, 85, 84, 80, 85, 88, 90, 97, 99, 85, 80, 83, 85, 89, 86, 92, 95, 98.
17. 88, 57, 87, 45, 66, 46, 78, 57, 67, 65, 44, 68, 90, 60, 58, 67, 45, 72, 51, 46, 75, 58.
18. 23, 26, 14, 25, 12, 11, 15, 17, 22, 19, 13, 15, 11, 17, 13, 10, 20, 16, 18, 13, 11, 16, 19, 21, 14, 24, 15, 18, 10, 13, 15.
19. 97, 88, 125, 132, 110, 89, 140, 128, 96, 68, 72, 150, 112, 128, 100, 104, 66, 75, 95, 85, 124.

20. 13.4, 13.3, 11.0, 7.6, 6.8, 11.0, 14.5, 15.5, 12.6, 13.2, 17.5, 19.3, 17.8, 13.4, 12.5, 15.2, 13.0, 17.0, 14.0, 18.0, 16.7.

21. 13.2, 15.6, 14.5, 17.5, 12.5, 17.5, 19.0, 14.0, 13.5, 12.4, 12.2, 17.5, 16.4, 12.8, 16.7, 13.4, 17.9, 16.5, 19.6, 14.6, 15.6, 15.4, 16.8, 13.5, 18.5, 19.0.

22. 51, 75, 54, 25, 43, 134, 61, 122, 25, 100, 109, 68, 38, 83, 75, 47, 20, 51, 50, 98.

23. 6, 2, 9, 5, 3, 9, 6, 1, 4, 0, 6, 8, 4, 0, 7, 2, 9, 4, 7, 3, 6, 4, 9, 4, 2, 0, 6, 0, 8, 3, 6, 0, 4, 7, 0, 9, 5, 3.

24. 25.56, 35.27, 73.25, 45.50, 24.45, 12.35, 13.66, 12.45, 17.65, 15.83, 14.32, 17.58, 36.43, 15.51, 14.55, 42.26, 16.72, 13.35, 16.57, 27.45, 22.75, 12.78.

25. 126, 135, 129, 100, 88, 124, 75, 86, 110, 145, 100, 98, 100, 92, 89, 120, 152, 100, 99, 102, 107, 110, 96, 85, 102, 100, 95, 80, 90, 100, 105.

CHAPTER X

THE ARITHMETIC MEAN, FROM A FREQUENCY DISTRIBUTION

Explanation. — The *arithmetic mean* is a name applied to a calculated average of a series of scores. This chapter shows how it is obtained from a frequency distribution, with a result which corresponds to the common average of a simple series. It is represented by the letter M, and is found by the use of the formula

$$M = \text{Estimated Mean} \pm \frac{\Sigma fd}{N} \times \text{size of interval.}$$

The parts of which are explained as follows:

Estimated mean, the mid-point of the mid-interval.

Σfd, the sum of the frequency deviations.

N, the number of cases.

The *size of the interval* is determined at sight.

The sign $\pm$ (plus-minus) is to be changed to either plus or minus, according to the sign of Σfd in each problem.

ILLUSTRATIVE PROBLEM. — Find the arithmetic mean:

INT.	f
75–79	2
70–74	6
65–69	5
60–64	9
55–59	15
50–54	12
45–49	3
40–44	2
35–39	2

Solution:

INT.	f	d	fd	
75–79	2	+4	+8	
70–74	6	+3	+18	
65–69	5	+2	+10	
60–64	9	+1	+9	(+45)
55–59	15	0		
50–54	12	−1	−12	
45–49	3	−2	−6	
40–44	2	−3	−6	
35–39	2	−4	−8	(−32)
	N 56		+13	(Σfd)

(Formula)

$$M = \text{Est. } M \pm \frac{\Sigma fd}{N} \times \text{int.}$$

$$M = 57.5 + \tfrac{13}{56} \times 5$$

$$M = 57.5 + \tfrac{65}{56}$$

$$M = 57.5 + 1.16$$

$$M = 58.66 \quad \text{(Answer)}$$

Outline of procedure. — To find the arithmetic mean:

(*a*) Arrange the data in a frequency distribution, and find N.

(*b*) Find the mid-interval, and mark it with a horizontal line above and below.

(*c*) Find the *estimated mean* (the mid-point of the mid-interval).

(*d*) Mark in the deviations, in a column headed "*d*". The mid-interval always has a deviation of zero; the interval just above it, +1; the next, +2; then +3, etc. The intervals below, counting from the mid-interval downward, will have deviations of −1, −2, −3, etc.

(*e*) Multiply each frequency by its corresponding deviation, and place the products in a column headed "*fd*". All *fd* measures above the mid-interval will carry the plus sign; those below, the minus sign.

(*f*) Find the algebraic sum of the *fd* columns.* This will be Σfd (the sum of the frequency deviations).

(*g*) Multiply Σfd by the size of the interval, and divide the result by N, as indicated by the formula.

* It will be found most convenient to first add the plus numbers, then the minus; then to add these two sums algebraically.

(*h*) Add to or subtract from the estimated mean, depending on the sign of Σfd. The result is the arithmetic mean.

ILLUSTRATIVE PROBLEM. — Find arithmetic mean:

INT.	*f*
90–99	1
80–89	3
70–79	5
60–69	6
50–59	11
40–49	10
30–39	6
20–29	4
10–19	2
0–9	2

Solution:

INT.	*f*	*d*	*fd*	
90–99	1	+4	+4	
80–89	3	+3	+9	
70–79	5	+2	+10	
60–69	6	+1	+6	(+29)
50–59	11	0		
40–49	10	−1	−10	
30–39	6	−2	−12	
20–29	4	−3	−12	
10–19	2	−4	−8	
0–9	2	−5	−10	(−52)
	N 50			−23 (Σfd)

(Formula) $M = \text{Est. } M \pm \frac{\Sigma fd}{N} \times \text{int.}$

$M = 55.0 - \frac{23}{50} \times 10$

$M = 55.0 - \frac{230}{50}$

$M = 55.0 - 4.6$

$M = 50.4$ (Answer)

ILLUSTRATIVE PROBLEM. —

INT.	*f*	*d*	*fd*	
160–179	2	+4	+8	
140–159	0	+3	+0	
120–139	4	+2	+8	
100–119	8	+1	+8	(+24)
80–99	13	0		
60–79	10	−1	−10	
40–59	3	−2	−6	
20–39	2	−3	−6	(−22)
	N 42			+ 2 (Σfd)

$M = \text{Est. } M \pm \frac{\Sigma fd}{N} \times \text{int.}$

$M = 90.0 + \frac{2}{42} \times 20$

$M = 90.0 + \frac{40}{42}$

$M = 90.0 + .9$

$M = 90.9$ (Answer)

PROBLEMS TO SOLVE

Find the arithmetic mean:

1.

INT.	f
120–129	1
110–119	2
100–109	3
90–99	8
80–89	6
70–79	6
60–69	13
50–59	6
40–49	3
30–39	2
	N 50

2.

INT.	f
150–159	3
140–149	4
130–139	12
120–129	7
110–119	6
100–109	13
90–99	4
80–89	0
70–79	1
	N 50

3.

INT.	f
90–99	2
80–89	0
70–79	0
60–69	3
50–59	5
40–49	10
30–39	3
20–29	0
10–19	0
0–9	2

4.

INT.	f
40–44	2
35–39	6
30–34	9
25–29	5
20–24	15
15–19	12
10–14	3
5–9	2
0–4	2

5.

INT.	f
95–99	3
90–94	5
85–89	8
80–84	12
75–79	15
70–74	20
65–69	17
60–64	10
55–59	5
50–54	3
45–49	2

6.

INT.	f
45–49	3
40–44	7
35–39	5
30–34	29
25–29	32
20–24	16
15–19	15
10–14	4
5–9	2
0–4	1

7.

INT.	f
90–99	1
80–89	3
70–79	5
60–69	10
50–59	11
40–49	6
30–39	4

8.

INT.	f
120–129	7
110–119	3
100–109	10
90–99	15
80–89	16
70–79	18
60–69	25
50–59	30
40–49	15
30–39	5
20–29	2

9.

INT.	f
80–81	5
78–79	7
76–77	8
74–75	9
72–73	14
70–71	10
68–69	8
66–67	5
64–65	5
62–63	2
60–61	2

10.

INT.	f
140–149	12
130–139	6
120–129	6
110–119	15
100–109	23
90–99	69
80–89	31
70–79	7
60–69	15
50–59	4
40–49	3
30–39	9

11.

INT.	f
95–99	3
90–94	5
85–89	14
80–84	20
75–79	27
70–74	7
65–69	28
60–64	18
55–59	20
50–54	8

12.

INT.	f
70–74	2
65–69	4
60–64	13
55–59	12
50–54	6
45–49	10
40–44	7
35–39	10
30–34	13
25–29	15
20–24	8

13.

INT.	f
200–209	2
190–199	0
180–189	4
170–179	6
160–169	4
150–159	3
140–149	2
130–139	1
120–129	0
110–119	2
100–109	1

14.

INT.	f
30–31	1
28–29	2
26–27	3
24–25	3
22–23	5
20–21	3
18–19	3
16–17	2
14–15	0
12–13	1
10–11	2

15.

INT.	f
200–214	2
185–199	5
170–184	3
155–169	9
140–154	16
125–139	20
110–124	14
95–109	8
80–94	10
65–79	7
50–64	3
35–49	3

16.

INT.	f
95–99	2
90–94	4
85–89	3
80–84	9
75–79	13
70–74	20
65–69	16
60–64	11
55–59	5
50–54	7
45–49	6
40–44	4

17.

INT.	f
140–149	5
130–139	5
120–129	10
110–119	15
100–109	25
90–99	60
80–89	30
70–79	12
60–69	5
50–59	4
40–49	9
30–39	3

18.

INT.	f
85–89	2
80–84	3
75–79	1
70–74	6
65–69	10
60–64	16
55–59	25
50–54	15
45–49	9
40–44	5
35–39	7
30–34	1

19.

INT.	f
100–104	2
95–99	0
90–94	5
85–89	12
80–84	15
75–79	21
70–74	20
65–69	16
60–64	4
55–59	3
50–54	2

20.

INT.	f
95–99	3
90–94	5
85–89	14
80–84	20
75–79	27
70–74	7
65–69	28
60–64	18
55–59	20
50–54	8

21.

INT.	f
100–109	2
90–99	12
80–89	16
70–79	22
60–69	12
50–59	9
40–49	11
30–39	6
20–29	7
10–19	1
0–9	2

22.

INT.	f
90–94	10
85–89	15
80–84	16
75–79	16
70–74	20
65–69	15
60–64	14
55–59	10
50–54	4

23.

INT.	f
72–75	2
68–71	4
64–67	13
60–63	12
56–59	6
52–55	10
48–51	7
44–47	10
40–43	13
36–39	15
32–35	8

24.

INT.	f
95–99	5
90–94	7
85–89	8
80–84	12
75–79	13
70–74	18
65–69	15
60–64	12
55–59	3
50–54	7

25.

INT.	f
90–92	1
87–89	6
84–86	6
81–83	7
78–80	8
75–77	13
72–74	16
69–71	11
66–68	5
63–65	2

26.

INT.	f
55–59	2
50–54	6
45–49	12
40–44	7
35–39	13
30–34	14
25–29	11
20–24	10
15–19	8
10–14	8
5–9	5
0–4	4

27.

INT.	f
190–199	5
180–189	8
170–179	10
160–169	14
150–159	43
140–149	40
130–139	32
120–129	27
110–119	14
100–109	7

28.

INT.	f
80–81	5
78–79	7
76–77	8
74–75	9
72–73	14
70–71	10
68–69	8
66–67	5
64–65	5
62–63	2
60–61	2

29.

INT.	f
98–99	2
96–97	2
94–95	0
92–93	6
90–91	10
88–89	21
86–87	18
84–85	11
82–83	3
80–81	2

30.

INT.	f
31	1
30	2
29	4
28	6
27	12
26	8
25	8
24	3
23	3
22	2
21	1

31.

INT.	f
95	1
94	0
93	4
92	5
91	3
90	12
89	14
88	10
87	7
86	8
85	3
84	2
83	1

32.

INT.	f
180–199	11
160–179	16
140–159	27
120–139	39
100–119	22
80–99	17
60–79	10
40–59	8

33.

INT.	f
90–104	1
75–89	8
60–74	20
45–59	29
30–44	22
15–29	18
0–14	2

34.

INT.	f
95–99	2
90–94	1
85–89	5
80–84	12
75–79	14
70–74	20
65–69	15
60–64	16
55–59	9
50–54	4
45–49	1
40–44	1

35.

INT.	f
87–89	2
84–86	1
81–83	3
78–80	6
75–77	9
72–74	11
69–71	7
66–68	5
63–65	2
60–62	3
57–59	1

36.

INT.	f
200–219	9
180–199	12
160–179	15
140–159	3
120–139	6
100–119	10
80–99	4
60–79	8
40–59	20
20–39	2
0–19	1

37.

INT.	f
95	2
94	1
93	2
92	4
91	5
90	9
89	12
88	11
87	2
86	1
85	1

38.

INT.	f
110–119	5
100–109	8
90–99	12
80–89	20
70–79	17
60–69	30
50–59	26
40–49	10
30–39	8
20–29	7
10–19	3
0–9	4

39.

INT.	f
90–99	7
80–89	8
70–79	12
60–69	20
50–59	29
40–49	21
30–39	17
20–29	15
10–19	11
0–9	10

40.

INT.	f
80	2
79	1
78	3
77	0
76	8
75	12
74	10
73	6
72	2
71	3
70	2
69	1

41.

INT.	f
150–164	5
135–149	10
120–134	15
105–119	30
90–104	20
75–89	10
60–74	5
45–59	5

42.

INT.	f
160–169	3
150–159	5
140–149	10
130–139	15
120–129	20
110–119	20
100–109	13
90–99	10
80–89	9
70–79	4
60–69	1

43.

INT.	f
180–189	3
170–179	4
160–169	5
150–159	5
140–149	8
130–139	10
120–129	16
110–119	12
100–109	12
90–99	8
80–89	10
70–79	7
60–69	4
50–59	0
40–49	3

44.

INT.	f
95–99	4
90–94	5
85–89	5
80–84	7
75–79	8
70–74	12
65–69	14
60–64	15
55–59	10
50–54	8
45–49	0
40–44	7
35–39	5
30–34	0
25–29	3
20–24	2

45.

INT.	f
66–67	2
64–65	2
62–63	4
60–61	3
58–59	0
56–57	7
54–55	0
52–53	8
50–51	9
48–49	15
46–47	10
44–45	14
42–43	2
40–41	2
38–39	0
36–37	4

46.

INT.	f
87–89	1
84–86	0
81–83	4
78–80	0
75–77	3
72–74	8
69–71	8
66–68	9
63–65	14
60–62	12
57–59	10
54–56	8
51–53	7
48–50	7
45–47	4
42–44	0
39–41	3

47.

INT.	f
88–91	3
84–87	7
80–83	8
76–79	8
72–75	9
68–71	10
64–67	12
60–63	15
56–59	12
52–55	7
48–51	5
44–47	5
40–43	0
36–39	4
32–35	0
28–31	3
24–27	1

48.

INT.	f
360–379	8
340–359	9
320–339	16
300–319	10
280–299	18
260–279	16
240–259	10
220–239	8
200–219	12
180–199	9
160–179	4
140–159	0
120–139	3
100–119	2

49.

INT.	f
165–169	2
160–164	0
155–159	4
150–154	5
145–149	5
140–144	6
135–139	8
130–134	10
125–129	18
120–124	20
115–119	15
110–114	15
105–109	12
100–104	15
95–99	14
90–94	7
85–89	8
80–84	4
75–79	6
70–74	8
65–69	5
60–64	3
55–59	0
50–54	0
45–49	2

50.

INT.	f
190–199	3
180–189	4
170–179	6
160–169	4
150–159	0
140–149	3
130–139	0
120–129	6
110–119	8
100–109	12
90–99	15
80–89	22
70–79	28
60–69	34
50–59	30
40–49	16
30–39	12
20–29	10
10–19	8
0–9	3

51.

INT.	f
90–92	4
87–89	5
84–86	8
81–83	0
78–80	3
75–77	0
72–74	4
69–71	7
66–68	9
63–65	10
60–62	15
57–59	18
54–56	16
51–53	12
48–50	15
45–47	20
42–44	16
39–41	13
36–38	10
33–35	8
30–32	0
27–29	7
24–26	2

52. Arrange the following scores in a frequency distribution, in intervals of 5, and find the arithmetic mean:

63, 72, 46, 38, 63, 82, 50, 57, 36, 40, 42, 67, 78, 48, 66, 88, 52, 31, 62, 72, 46, 62, 37, 27, 50, 56, 58, 66, 52, 52, 77, 42, 70, 41, 65, 52, 65, 46, 55, 63, 42, 75, 66, 58, 42, 51, 73, 45, 82, 84, 42, 82.

53. Distribute in intervals of 10 and find the arithmetic mean:

73, 88, 42, 36, 66, 75, 54, 22, 8, 96, 53, 62, 55, 40, 55, 75, 63, 44, 28, 85, 36, 32, 73, 27, 15, 82, 44, 16, 50, 60, 33, 65, 36, 67, 24, 37, 55, 25, 50, 75, 43, 82, 34, 62, 25, 80, 55, 37, 52, 68, 85, 29, 62, 45, 75.

54. Distribute in intervals of 15 and find the arithmetic mean:

76, 52, 118, 73, 115, 40, 16, 107, 75, 80, 50, 73, 100, 48, 88, 28, 18, 73, 92, 35, 74, 90, 45, 12, 47, 10, 65, 100, 128, 47, 130, 35, 105, 70, 95.

55. Distribute in intervals of 2 and find the arithmetic mean:

42, 37, 45, 35, 28, 40, 49, 38, 39, 42, 47, 30, 51, 48, 40, 40, 41, 46, 45, 32, 33, 39, 35, 30, 36, 30, 44, 35, 37, 50, 39, 29, 42, 36, 45, 36, 42, 46, 39, 47, 35, 40, 38, 42.

REFERENCES ON THE ARITHMETIC MEAN

Chaddock, *Principles and Methods of Statistics*, ch. VI.
Garrett, *Statistics in Psychology and Education*, p. 8.
Holzinger, *Statistical Methods for Students in Education*, ch. VI.
Kelley, *Statistical Method*, p. 45.
King, *Elements of Statistical Method*, p. 132.
Lincoln, *Beginnings in Educational Measurement*, p. 37.
McCall, *How to Measure in Education*, p. 366.
Odell, *Educational Statistics*, p. 66.
Otis, *Statistical Methods in Educational Measurement*, p. 17.
Thurstone, *The Fundamentals of Statistics*, ch. X.

CHAPTER XI

THE MID-SCORE

Explanation. — The mid-score is the middle one of a series of numbers arranged in order. It is a good measure of central tendency, and is often more significant than the common average. The ease with which it is found tends to eliminate error and to save time.

To find the mid-score of any series of numbers, arrange them in order, and count to the middle one.

ILLUSTRATIVE PROBLEM. —

Find the mid-score: 80, 92, 96, 73, 74, 76, 82, 85, 88

Solution:

96
92
88
85
82 (mid-score)
80
76
74
73

ILLUSTRATIVE PROBLEM. — Find the mid-score:

85, 92, 97, 96, 97, 85, 80, 83, 88, 76, 79.

Solution:

97
97
96
92
88
85 (mid-score)
85
83
80
79
76

ILLUSTRATIVE PROBLEM. —

Find the mid-score:

	Solution:	
85	113	
99	100	
100	99	
113	99	
99	95	94, mid-score
85	93	
95	88	
88	85	
82	85	
93	82	

In the foregoing problem there are ten cases, so there are really two mid-scores, 93 and 95. So we simply average them, and use 94 as the mid-score.

ILLUSTRATIVE PROBLEM. —

Find the mid-score:

	Solution:	
4	28	
18	24	
9	20	
19	20	
24	19	
19	19	18.5, mid-score
20	18	
15	18	
18	15	
0	9	
20	4	
28	0	

ILLUSTRATIVE PROBLEM. —

Find the mid-score:

	Solution:	
37	45	
37	42	
40	41	
45	40	
36	40	
42	37	37, mid-score
36	37	
41	36	
35	36	
32	35	
35	35	
40	32	

PROBLEMS TO SOLVE

Find the mid-score:

1. 87, 96, 83, 94, 89, 99, 82, 88, 95, 90, 84.
2. 60, 75, 71, 65, 73, 58, 74, 70, 62, 50, 68, 73, 66.
3. 35, 53, 45, 50, 40, 43, 50, 56, 39, 42, 49, 55, 30, 46, 30.
4. 90, 96, 100, 83, 89, 96, 97, 81, 88, 96, 98, 94, 81, 80, 86.
5. 10, 18, 15, 18, 20, 15, 17, 16, 19, 12, 12, 17, 15, 19, 9.
6. 20, 28, 30, 20, 16, 26, 35, 27, 20, 30, 26, 18, 26, 34, 36, 25, 19.
7. 52, 39, 47, 51, 56, 36, 30, 57, 50, 42, 43, 58, 40, 33, 35, 48, 54.
8. 90, 81, 86, 82, 81, 85, 90, 91, 96, 93, 88, 98, 89, 90, 85, 86, 91, 94, 83.
9. 66, 68, 72, 50, 67, 64, 70, 71, 72, 58, 69, 62, 71, 66, 71, 57, 55, 60, 63, 69, 67.
10. 38, 44, 48, 28, 20, 40, 42, 30, 47, 20, 24, 34, 26, 36, 41, 43, 29, 39, 45, 32, 23.
11. 93, 98, 86, 95, 97, 74, 93, 80, 100, 89, 75, 96, 90, 95, 92, 84, 92, 94, 82, 90, 87.

12. 70, 84, 67, 58, 88, 63, 52, 74, 88, 64, 60, 55, 74, 66, 80, 76, 90, 62, 78, 69, 86, 89, 68.

13. 117, 124, 100, 110, 130, 103, 95, 120, 118, 123, 112, 128, 114, 100, 120, 129, 106, 99, 96, 104, 109, 100, 96, 126, 98.

14. 61, 57, 62, 56, 53, 50, 54, 64, 60, 54, 48, 65, 47, 60, 62, 56, 61, 60, 58, 63, 61, 52, 62, 58, 62.

15. 80, 66, 63, 71, 76, 72, 73, 89, 81, 68, 67, 55, 60, 55, 82, 62, 82, 57, 70, 80, 58, 86, 74, 78, 84.

16. 16, 17, 17, 16, 13, 10, 8, 10, 12, 16, 12, 8, 10, 15, 17, 15, 16, 13, 15, 10, 14, 16, 12, 15, 16, 10, 16.

17. 80, 90, 94, 83, 70, 58, 85, 87, 76, 88, 85, 92, 79, 68, 72, 56, 71, 87, 93, 86, 80, 91, 70, 60, 92, 84, 63.

18. 35, 29, 43, 15, 21, 8, 22, 34, 46, 33, 26, 19, 9, 24, 36, 41, 20, 10, 30, 42, 27, 16, 32, 40, 17, 12, 38, 45, 8.

19. 16, 14, 12, 18, 25, 14, 7, 10, 5, 15, 21, 18, 15, 8, 6, 19, 5, 13, 9, 12, 16, 14, 20, 17, 14, 15, 7, 6, 23, 24.

20. 12, 9, 4, 2, 5, 0, 4, 9, 15, 10, 8, 5, 1, 6, 4, 5, 9, 7, 11, 15, 13, 10, 12, 7, 3, 10, 14, 1, 3, 2.

21. 66, 57, 51, 67, 76, 69, 58, 50, 64, 72, 68, 59, 71, 56, 46, 70, 74, 61, 54, 62, 70, 53, 48, 56, 73, 50.

22. 46, 30, 24, 44, 49, 47, 40, 40, 36, 28, 26, 48, 45, 38, 46, 48, 30, 42, 48, 49.

23. 134, 129, 145, 136, 144, 130, 150, 124, 115, 138, 148, 140, 129, 142, 145, 127, 132, 140, 120, 146, 115, 116, 118.

24. 92, 120, 127, 140, 110, 122, 147, 87, 118, 62, 96, 133, 73, 113, 138, 80, 100, 141, 60, 42, 150, 148, 142, 36, 54.

25. 95, 85, 78, 67, 88, 77, 75, 99, 80, 70, 66, 94, 84, 71, 61, 93, 90, 76, 54, 98, 60, 48, 68, 83, 91, 73, 65, 72, 92, 97, 81, 62, 96, 86, 46, 41, 52, 47, 89, 64, 59, 51, 44, 40, 87, 74, 63, 57, 50, 53, 55, 82, 69, 45, 42, 79, 56, 49, 43, 58, 100.

26. 15, 23, 25, 47, 58, 50, 30, 20, 14, 70, 65, 58, 48, 25, 12, 10, 8, 14, 10, 18, 15, 18, 25, 10, 37, 25, 45, 40, 53, 50, 41, 25, 57, 58, 25, 67, 59, 70, 67, 58, 70, 60, 69, 75, 70, 43, 38, 24, 38, 10, 7, 10, 10.

27. 95, 90, 86, 97, 93, 100, 87, 100, 93, 88, 85, 96, 86, 80, 93, 100, 89, 86, 96, 90, 88, 94, 88, 96, 93, 100, 88, 83, 93, 81, 84, 70, 83, 88, 83, 90, 80, 75, 95, 75, 100, 74, 83, 75, 82, 93, 85, 96, 82, 93.

28. 172, 182, 175, 166, 186, 145, 187, 173, 158, 185, 167, 181, 170, 185, 175, 168, 180, 167, 184, 170, 150, 187, 170, 165, 180, 174, 182, 178, 165, 174, 164, 169, 162, 173, 166, 179, 184, 180, 176, 165, 175, 160, 178, 156, 180, 184, 178, 168, 177, 179, 170, 168.

29. 206, 210, 198, 207, 185, 173, 209, 202, 208, 212, 206, 184, 205, 175, 207, 200, 209, 207, 211, 198, 178, 207, 190, 177, 160, 207, 179, 210, 180, 204, 210, 205, 184, 203, 207, 178, 207, 168, 208, 170, 160, 195, 206, 165, 207, 187, 197, 209, 207, 209, 200.

30. 94, 82, 98, 84, 88, 98, 70, 93, 80, 97, 87, 98, 80, 83, 90, 80, 92, 98, 80, 95, 83, 98, 85, 80, 91, 97, 88, 80, 82, 94, 72, 97, 90, 97, 81, 90, 81, 89, 80, 95, 98, 80, 75.

31. 15, 10, 19, 17, 19, 15, 14, 19, 17, 13, 18, 16, 10, 19, 17, 19, 12, 17, 18, 10, 17, 14, 19, 10, 19, 8, 16, 19, 18, 12, 18, 10, 19, 15, 10, 19.

32. 117, 110, 128, 100, 120, 115, 137, 124, 100, 130, 118, 102, 135, 120, 129, 110, 134, 114, 138, 118, 105, 127, 113, 130, 109, 135, 125, 107, 134, 137, 128, 118, 126, 129, 130.

33. 15.5, 10.8, 16.8, 10.2, 17.6, 16.0, 19.0, 16.5, 14.6, 18.2, 10.5, 16.0, 12.9, 17.2, 16.4, 18.5, 16.0, 18.0, 13.2, 17.0, 10.8, 18.3, 16.8, 17.0, 15.7, 17.3, 15.0, 15.9, 16.5.

CHAPTER XII

THE MEDIAN

Explanation. — The *median* is the middle measure; or, more exactly, the calculated position of the middle case. We have seen (Chapter XI) that in a simple series of scores we may find the middle case, or mid-score, by counting to it. When we have our scores arranged in a frequency distribution, the individual scores have lost their identity, so we cannot count them as separate units. For this reason the calculation of the middle case becomes somewhat more complex. The median always occurs in the mid-interval, for the mid-interval has been defined as the interval containing the middle case. The position of this case is located by a process of counting, *as if the frequencies in each interval were equally distributed through it.*

Outline of procedure. — After the frequency distribution is made, and the intervals are completed,

(*a*) find the total of the frequencies (N),

(*b*) divide N by 2, $\left(\frac{N}{2}\right)$,

(*c*) find the mid-interval,

(*d*) count the number of frequencies *above* the mid-interval,

(*e*) subtract the number of frequencies above the mid-interval from $\frac{N}{2}$ (this gives the number *yet to be used*),

(*f*) divide the number *yet to be used* by the frequency

of the mid-interval (by expressing it as a fraction),

(*g*) multiply this fraction by the size of the interval,

(*h*) subtract the product from the top of the mid-interval (which will be the number with which the next interval above begins). The result will be the median (*Md.*).

$$\text{Md.} = \text{Top of mid-int.} - \frac{f \text{ yet to be used}}{f \text{ of mid-int.}} \times \text{size of int.}$$

ILLUSTRATIVE PROBLEM. —

Find the median:

INT.	*f*
85–89	3
80–84	4
75–79	8
70–74	12
65–69	7
60–64	5
55–59	4
50–54	2
45–49	1
N	46

Solution:

INT.	*f*	
85–89	3	
80–84	4	
75–79	8	(15)
70–74	12	
65–69	7	
60–64	5	
55–59	4	
50–54	2	
45–49	1	
N	46	

$Md. = 75 - \frac{8}{12} \times 5$

$Md. = 75 - \frac{40}{12}$

$Md. = 75.0 - 3.3$

$Md. = 71.7$ (Ans.)

Analysis of problem by steps shown in outline of procedure:

(*a*) $N = 46$

(*b*) $\frac{N}{2} = 23$

(*c*) Mid-interval, 70–74

(*d*) No. of measures above mid-interval, 15

(*e*) No. of measures yet to be used, $23 - 15 = 8$

(*f, g, h*) $75 - \frac{8}{12} \times 5 = 71.7$

Graphic explanation of practice problem on page 62. —

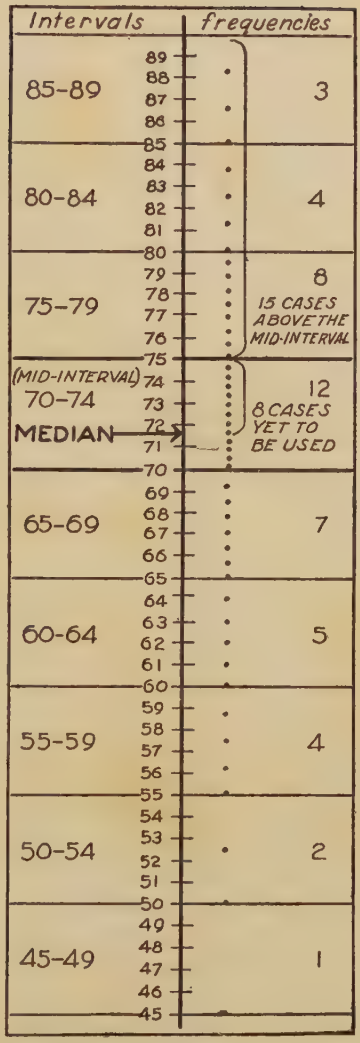
$$N = 46$$

$$\frac{N}{2} = 23$$

Hence the median will be the 23rd case.
Mid-interval, 70–74.
Top of mid-interval, 75.
Total frequencies above the mid-interval, 15.
Number yet to be used to find the middle case, 8.

Solution:

$$Md. = 75 - \tfrac{8}{12} \times 5$$
$$Md. = 75 - \tfrac{40}{12}$$
$$Md. = 75 - 3.3$$
$$Md. = 71.7 \text{ (Ans.)}$$



ILLUSTRATIVE PROBLEM. —

Find the median:

INT.	f
90–99	2
80–89	3
70–79	5
60–69	8
50–59	2
40–49	2
30–39	3
20–29	2
10–19	2
0–9	1
N	30

Solution:

INT.	f	
90–99	2	
80–89	3	
70–79	5	(10)
60–69	8	
50–59	2	
40–49	2	
30–39	3	
20–29	2	
10–19	2	
0–9	1	
N	30	

$Md. = 70 - \frac{5}{8} \times 10$

$Md. = 70 - \frac{50}{8}$

$Md. = 70.0 - 6.2$

$Md. = 63.8$ (Ans.)

Analysis of problem:

$N = 30$

$\frac{N}{2} = 15$

Mid-interval, 60–69

Top of mid-interval, 70

Total frequencies above mid-interval, 10

No. yet to be used, 5

Frequency of mid-interval, 8

Size of interval, 10

$Md. = 70 - \frac{5}{8} \times 10$

$Md. = 63.8$

ILLUSTRATIVE PROBLEM, *with solution.* —

INT.	f	
95	3	
94	8	$\frac{N}{2} = 20$
93	7	
92	0	Top of mid-interval, 91
91	0 (18)	Total f above, 18
90	4	No. yet to be used, $20 - 18 = 2$
89	3	Frequency of mid-interval, 4
88	6	Size of interval, 1
87	4	$Md. = 91 - \frac{2}{4} \times 1$
86	3	$Md. = 91.0 - 0.5$
85	2	$Md. = 90.5$ (Ans.)
N	40	

ILLUSTRATIVE PROBLEM, *with solution.* —

INT.	f	
90–92	8	
87–89	10	$\frac{N}{2} = 41.5$
84–86	12 (30)	
81–83	16	Top of mid-interval, 84
78–80	10	Total f above mid-interval, 30
75–77	8	No. yet to be used, $41.5 - 30 = 11.5$
72–74	9	Frequency of mid-interval, 16
69–71	7	Size of interval, 3
66–68	3	$Md. = 84 - \frac{11.5}{16} \times 3$
N	83	

$$Md. = 84 - \frac{34.5}{16}$$

$$Md. = 84.0 - 2.1$$

$$Md. = 81.9 \text{ (Ans.)}$$

ILLUSTRATIVE PROBLEM, *with solution.* —

INT.	f	
125–129	2	
120–124	0	
115–119	6	
110–114	8	(16)
105–109	10	
100–104	9	
95–99	7	
90–94	6	
85–89	2	
	N 50	

$\frac{N}{2} = 25$

$Md. = 110 - \frac{9}{10} \times 5$

$Md. = 110 - \frac{45}{10}$

$Md. = 110.0 - 4.5$

$Md. = 105.5$ (Ans.)

ILLUSTRATIVE PROBLEM, *with solution.* —

INT.	f	
48–49	1	
46–47	3	
44–45	4	
42–43	7	(15)
40–41	13	
38–39	14	
36–37	8	
34–35	2	
32–33	0	
30–31	3	
28–29	1	
	N 56	

$\frac{N}{2} = 28$

$Md. = 42 - \frac{13}{13} \times 2$

$Md. = 42.0 - 2.0$

$Md. = 40.0$ (Ans.)

PROBLEMS TO SOLVE

Find the median:

1.

INT.	*f*
95–99	2
90–94	3
85–89	5
80–84	7
75–79	10
70–74	12
65–69	8
60–64	9
55–59	7
50–54	3
	N 66

2.

INT.	*f*
90–99	1
80–89	4
70–79	3
60–69	8
50–59	11
40–49	18
30–39	10
20–29	7
10–19	6
0–9	4

3.

INT.	*f*
80–84	1
75–79	1
70–74	2
65–69	5
60–64	8
55–59	10
50–54	5
45–49	3
40–44	3
35–39	2

4.

INT.	*f*
95–99	2
90–94	3
85–89	5
80–84	4
75–79	5
70–74	8
65–69	7
60–64	3
55–59	7
50–54	8

5.

INT.	*f*
75–77	2
72–74	3
69–71	3
66–68	4
63–65	4
60–62	5
57–59	3
54–56	5
51–53	3
48–50	2

6.

INT.	*f*
120–121	2
118–119	4
116–117	6
114–115	8
112–113	5
110–111	7
108–109	5
106–107	3
104–105	3
102–103	2

7.

INT.	*f*
110–119	1
100–109	3
90–99	5
80–89	4
70–79	6
60–69	10
50–59	15
40–49	8
30–39	7
20–29	3
10–19	2

8.

INT.	*f*
100–109	2
90–99	1
80–89	2
70–79	4
60–69	1
50–59	6
40–49	0
30–39	8
20–29	3
10–19	2
0–9	1

9.

INT.	*f*
160–179	2
140–159	4
120–139	6
100–119	13
80–99	8
60–79	7
40–59	5
20–39	7
0–19	8

10.

INT.	f
220–239	3
200–219	7
180–199	8
160–179	4
140–159	6
120–139	6
100–119	2
80–99	0
60–79	2

11.

INT.	f
150–174	6
125–149	6
100–124	8
75–99	13
50–74	8
25–49	4
0–24	3

12.

INT.	f
90–94	3
85–89	7
80–84	8
75–79	7
70–74	10
65–69	8
60–64	7
55–59	6
50–54	4

13.

INT.	f
70–74	1
65–69	2
60–64	0
55–59	7
50–54	10
45–49	5
40–44	8
35–39	7
30–34	7
25–29	4
20–24	2

14.

INT.	f
78–80	2
75–77	3
72–74	5
69–71	8
66–68	2
63–65	3
60–62	7
57–59	3
54–56	2
51–53	3

15.

INT.	f
88–89	1
86–87	2
84–85	3
82–83	4
80–81	6
78–79	9
76–77	6
74–75	3
72–73	2
70–71	2

16.

INT.	f
90	2
89	3
88	6
87	7
86	10
85	8
84	5
83	4
82	3
81	0
80	2

17.

INT.	f
90–104	2
75–89	4
60–74	15
45–59	14
30–44	3
15–29	4
0–14	2

18.

INT.	f
175–199	2
150–174	3
125–149	10
100–124	18
75–99	8
50–74	5
25–49	4
0–24	3

19.

INT.	*f*
85	2
84	3
83	6
82	9
81	10
80	8
79	6
78	5
77	4
76	3
75	2

20.

INT.	*f*
110–111	4
108–109	7
106–107	9
104–105	10
102–103	6
100–101	4
98–99	7
96–97	5
94–95	8
92–93	5
90–91	3

21.

INT.	*f*
120–124	2
115–119	1
110–114	3
105–109	4
100–104	8
95–99	14
90–94	8
85–89	8
80–84	5
75–79	4
70–74	3

22.

INT.	*f*
90–94	5
85–89	3
80–84	9
75–79	3
70–74	10
65–69	8
60–64	6
55–59	5
50–54	6
45–49	6

23.

INT.	*f*
120–129	2
110–119	4
100–109	6
90–99	8
80–89	10
70–79	12
60–69	5
50–59	3
40–49	2
30–39	1

24.

INT.	*f*
90–104	2
75–89	3
60–74	5
45–59	10
30–44	8
15–29	6
0–14	4

25.

INT.	*f*
200–219	5
180–199	16
160–179	19
140–159	12
120–139	8
100–119	6

26.

INT.	*f*
400–449	3
350–399	7
300–349	15
250–299	10
200–249	10
150–199	8
100–149	7

27.

INT.	*f*
34–35	5
32–33	6
30–31	4
28–29	10
26–27	26
24–25	15
22–23	5
20–21	0
18–19	3
16–17	1

28.

INT.	f
80–89	4
70–79	9
60–69	20
50–59	10
40–49	8
30–39	7
20–29	6
10–19	4
0–9	2

29.

INT.	f
80–84	1
75–79	1
70–74	2
65–69	5
60–64	8
55–59	10
50–54	5
45–49	3
40–44	3
35–39	2

30.

INT.	f
110–119	5
100–109	8
90–99	7
80–89	5
70–79	8
60–69	7
50–59	6
40–49	9
30–39	5
20–29	4

31.

INT.	f
100–104	4
95–99	7
90–94	9
85–89	10
80–84	15
75–79	5
70–74	7
65–69	8
60–64	5

32.

INT.	f
120–121	2
118–119	4
116–117	8
114–115	6
112–113	5
110–111	7
108–109	3
106–107	5
104–105	2
102–103	3

33.

INT.	f
75–77	4
72–74	3
69–71	2
66–68	3
63–65	4
60–62	5
57–59	3
54–56	2
51–53	3
48–50	5

34.

INT.	f
70–74	1
65–69	2
60–64	0
55–59	7
50–54	10
45–49	5
40–44	8
35–39	7
30–34	2
25–29	7
20–24	4

35.

INT.	f
90–104	2
75–89	3
60–74	5
45–59	10
30–44	8
15–29	7
0–14	4

36.

INT.	f
45	3
44	2
43	6
42	9
41	10
40	8
39	7
38	5
37	5
36	3

37.

INT.	f
65–69	2
60–64	0
55–59	3
50–54	10
45–49	12
40–44	15
35–39	10
30–34	8
25–29	6
20–24	5
15–19	4
10–14	3
5–9	2
0–4	2

38.

INT.	f
30–31	1
28–29	2
26–27	3
24–25	3
22–23	4
20–21	4
18–19	5
16–17	10
14–15	11
12–13	8
10–11	6
8–9	7
6–7	4
4–5	3
2–3	1

39.

INT.	f
130–134	5
125–129	7
120–124	10
115–119	13
110–114	24
105–109	35
100–104	52
95–99	61
90–94	62
85–89	60
80–84	73
75–79	51
70–74	40
65–69	32
60–64	17

40.

INT.	f
180–184	3
175–179	5
170–174	7
165–169	9
160–164	11
155–159	17
150–154	15
145–149	9
140–144	8
135–139	6
130–134	1
125–129	5

41.

INT.	f
120–129	3
110–119	7
100–109	10
90–99	11
80–89	20
70–79	29
60–69	25
50–59	16
40–49	10
30–39	5
20–29	2
10–19	3
0–9	2

42.

INT.	f
95–99	1
90–94	1
85–89	0
80–84	6
75–79	4
70–74	18
65–69	7
60–64	11
55–59	20
50–54	7
45–49	6
40–44	9
35–39	0
30–34	3

43.

INT.	f
90–99	8
80–89	13
70–79	15
60–69	14
50–59	13
40–49	20
30–39	7
20–29	13
10–19	6
0–9	5

44.

INT.	f
80–84	5
75–79	3
70–74	6
65–69	10
60–64	19
55–59	15
50–54	7
45–49	9
40–44	0
35–39	4
30–34	3
25–29	1

45.

INT.	f
120–129	4
110–119	5
100–109	6
90–99	8
80–89	12
70–79	25
60–69	36
50–59	28
40–49	12
30–39	4
20–29	2

46.

INT.	f
200–219	5
180–199	6
160–179	4
140–159	8
120–139	12
100–119	25
80–99	36
60–79	28
40–59	12
20–39	4
0–19	2

47.

INT.	f
125–129	1
120–124	0
115–119	9
110–114	13
105–109	15
100–104	10
95–99	11
90–94	7
85–89	3
80–84	4
75–79	3
70–74	1

48.

INT.	f
110–119	2
100–109	4
90–99	9
80–89	7
70–79	13
60–69	20
50–59	19
40–49	14
30–39	8
20–29	5

49.

INT.	f
130–139	5
120–129	7
110–119	10
100–109	15
90–99	20
80–89	25
70–79	30
60–69	10
50–59	10
40–49	0
30–39	7
20–29	5

50.

INT.	f
140–144	1
135–139	3
130–134	7
125–129	10
120–124	12
115–119	14
110–114	10
105–109	8
100–104	6
95–99	7
90–94	5
85–89	1

51.

INT.	f
120–129	1
110–119	5
100–109	10
90–99	20
80–89	50
70–79	7
60–69	3
50–59	2
40–49	2

52.

INT.	f
75–79	3
70–74	4
65–69	5
60–64	9
55–59	11
50–54	17
45–49	16
40–44	14
35–39	8
30–34	3
25–29	0
20–24	1

53.

INT.	f
150–159	4
140–149	6
130–139	8
120–129	9
110–119	9
100–109	15
90–99	25
80–89	20
70–79	14
60–69	8
50–59	10
40–49	6
30–39	4

54.

INT.	f
125–129	1
120–124	2
115–119	5
110–114	9
105–109	11
100–104	16
95–99	13
90–94	9
85–89	7
80–84	0
75–79	5
70–74	3

55.

INT.	f
90–94	1
85–89	1
80–84	2
75–79	0
70–74	4
65–69	8
60–64	15
55–59	13
50–54	7
45–49	8
40–44	5
35–39	4
30–34	2

56.

INT.	f
68–69	4
66–67	5
64–65	5
62–63	6
60–61	7
58–59	9
56–57	10
54–55	12
52–53	12
50–51	10
48–49	14
46–47	8
44–45	3
42–43	3
40–41	2

57.

INT.	f
140–149	3
130–139	4
120–129	3
110–119	7
100–109	14
90–99	22
80–89	20
70–79	16
60–69	8
50–59	4
40–49	3
30–39	0
20–29	0
10–19	2
0–9	2

58.

INT.	f
117	4
116	6
115	6
114	9
113	18
112	20
111	24
110	27
109	20
108	16
107	12
106	10
105	7
104	7
103	4
102	3

59.

INT.	f
90–92	5
87–89	5
84–86	0
81–83	4
78–80	0
75–77	8
72–74	8
69–71	5
66–68	7
63–65	0
60–62	4
57–59	8
54–56	7
51–53	3
48–50	0
45–47	2

60.

INT.	f
88–91	4
84–87	3
80–83	7
76–79	8
72–75	15
68–71	16
64–67	12
60–63	17
56–59	12
52–55	4
48–51	3
44–47	8
40–43	9
36–39	16
32–35	23
28–31	14
24–27	7
20–23	3

61.

INT.	f
180–184	1
175–179	0
170–174	0
165–169	1
160–164	2
155–159	3
150–154	4
145–149	4
140–144	0
135–139	3
130–134	5
125–129	8
120–124	12
115–119	15
110–114	22
105–109	25
100–104	20
95–99	18
90–94	17
85–89	10
80–84	8
75–79	12
70–74	7
65–69	3
60–64	0
55–59	2
50–54	1

62.

INT.	f
88–89	3
86–87	2
84–85	4
82–83	8
80–81	10
78–79	10
76–77	8
74–75	7
72–73	12
70–71	10
68–69	9
66–67	4
64–65	7
62–63	8
60–61	15
58–59	18
56–57	22
54–55	24
52–53	18
50–51	16
48–49	12
46–47	10
44–45	8
42–43	8
40–41	3

63.

INT.	f
200–209	2
190–199	0
180–189	1
170–179	4
160–169	5
150–159	8
140–149	16
130–139	25
120–129	32
110–119	37
100–109	48
90–99	44
80–89	52
70–79	47
60–69	40
50–59	35
40–49	23
30–39	18
20–29	20
10–19	5
0–9	3

64. Distribute in intervals of 10 and find the median: 65, 42, 83, 25, 45, 32, 8, 45, 53, 40, 88, 40, 20, 32, 56, 30, 36, 58, 66, 32, 24, 15, 24, 22, 53, 56, 75, 63, 82, 37, 10, 8, 45, 50, 35, 63, 15, 65, 72.

65. Distribute in intervals of 5 and find the median:

55, 36, 57, 38, 30, 42, 65, 42, 25, 40, 50, 52, 70, 45, 47, 30, 52, 65, 55, 60, 75, 62, 35, 75, 62, 46, 39, 25, 40, 40, 55, 58, 46, 42, 35, 32, 48, 52, 60, 50, 56, 72, 73, 55, 70, 36, 42, 33, 46, 48, 35, 62, 35, 45, 32, 65, 63, 44, 42, 45, 53, 32, 50, 36, 72, 35, 42, 62, 54, 62, 55, 42, 50.

66. Distribute in intervals of 5 and find the median:

136, 122, 114, 144, 130, 163, 142, 102, 125, 166, 173, 150, 156, 153, 137, 122, 125, 107, 123, 143, 158, 135, 120, 114, 122, 110, 152, 146, 170, 141, 147, 120, 110, 133, 145, 164, 129, 105, 126, 142, 155, 112, 132, 150, 135, 165, 125, 154, 136, 140, 147, 113, 136, 138, 135, 159, 162, 127, 125, 136, 148, 114.

67. Distribute in intervals of 2 and find the median:

52, 36, 47, 41, 50, 58, 49, 38, 32, 45, 43, 46, 59, 46, 42, 46, 47, 40, 36, 48, 42, 43, 52, 39, 52, 42, 36, 33, 50, 49, 58, 44, 41, 50, 53, 37, 44, 36, 45, 38, 33, 52, 46, 48, 52, 40, 49, 37, 49, 53, 37, 43, 47, 42, 44, 50, 39, 49, 58, 42, 46, 37, 42, 43, 48, 45, 52, 40, 32, 41, 47, 50, 58, 43, 48, 41, 46, 50, 38, 32, 38, 45, 36, 47, 40, 45, 52, 37, 40, 41, 46, 50, 58, 49, 43, 39, 44, 50, 54, 50.

ANOTHER WAY TO FIND THE MEDIAN

Instead of counting down from the top of the distribution, we may count up from the bottom to the middle case, with the same result. In this case the steps are as follows:

(*a*) Find the total of the frequencies (N).

(*b*) Divide N by 2, $\left(\frac{N}{2}\right)$.

(*c*) Find the mid-interval.

(*d*) Count the number of frequencies *below* the mid-interval.

(*e*) Subtract this number from $\frac{N}{2}$ (this gives the number *yet to be used*).

(*f*) Divide the number *yet to be used* by the frequency of the mid-interval (by expressing it as a fraction).

(*g*) Multiply this fraction by the size of the interval.

(*h*) Add the product to the beginning of the mid-interval. The result will be the median.

$$\text{Md.} = \text{Bottom of mid-int.} + \frac{f \text{ yet to be used}}{f \text{ of mid-interval}} \times \text{size of int.}$$

ILLUSTRATIVE PROBLEM, *solved by both methods.* —

INT.	*f*	
90–94	2	
85–89	3	
80–84	3	
75–79	4	
70–74	2	
65–69	4	
60–64	6	
55–59	9	
50–54	8	(41)
45–49	10	
40–44	10	(33)
35–39	9	
30–34	7	
25–29	3	
20–24	2	
15–19	2	
	N 84	

$\frac{N}{2} = 42$

First method:

Md. $= 50 - \frac{1}{10} \times 5$

Md. $= 50 - .5$

Md. $= 49.5$ (Ans.)

Second method:

Md. $= 45 + \frac{8}{10} \times 5$

Md. $= 45 + \frac{4}{10}^{5}$

Md. $= 45 + 4.5$

Md. $= 49.5$ (Ans.)

ILLUSTRATIVE PROBLEM, *with solution.* —

INT.	f	
60–64	1	
55–59	0	
50–54	2	
45–49	7	
40–44	8	
35–39	13	
30–34	17	
25–29	20	(42)
20–24	10	
15–19	9	
10–14	2	
5–9	1	
	N 90	

$N = 90$

$\frac{N}{2} = 45$

Total f below mid-interval, 42

No. yet to be used, $45 - 42 = 3$

Frequency of mid-interval, 17

Bottom of mid-interval, 30

Size of interval, 5

$Md. = 30 + \frac{3}{17} \times 5$

$Md. = 30 + \frac{15}{17}$

$Md. = 30 + .8$

$Md. = 30.8$ (Ans.)

ILLUSTRATIVE PROBLEM, *with solution.* —

INT.	f	
100–109	4	
90–99	5	
80–89	5	
70–79	10	
60–69	15	
50–59	20	
40–49	16	(41)
30–39	10	
20–29	8	
10–19	5	
0–9	2	
	N 100	

$\frac{N}{2} = 50$

Size of interval, 10

$Md. = 50 + \frac{9}{20} \times 10$

$Md. = 50 + 4.5$

$Md. = 54.5$ (Ans.)

PROBLEMS TO SOLVE

Find the median:

68.

INT.	*f*
90–99	1
80–89	0
70–79	3
60–69	2
50–59	4
40–49	10
30–39	4
20–29	3
10–19	1
0–9	2
	30

69.

INT.	*f*
95–99	2
90–94	1
85–89	3
80–84	2
75–79	5
70–74	4
65–69	4
60–64	2
55–59	1
50–54	1
45–49	2
	27

70.

INT.	*f*
80–99	1
60–79	2
40–59	10
20–39	8
0–19	3
	24

71.

INT.	*f*
90–104	1
75–89	6
60–74	4
45–59	3
30–44	6
15–29	4
0–14	2
	26

72.

INT.	*f*
90–99	1
80–89	2
70–79	2
60–69	4
50–59	5
40–49	3
30–39	2
20–29	2
10–19	1
	22

73.

INT.	*f*
88–89	1
86–87	2
84–85	2
82–83	4
80–81	4
78–79	4
76–77	2
74–75	1
	20

74.

INT.	f
200–219	2
180–199	4
160–179	5
140–159	10
120–139	8
100–119	4
80–99	3

75.

INT.	f
60–62	1
57–59	2
54–56	3
51–53	4
48–50	8
45–47	6
42–44	4
39–41	2

76.

INT.	f
52–53	2
50–51	3
48–49	5
46–47	7
44–45	5
42–43	3
40–41	2
38–39	2

77.

INT.	f
95–99	1
90–94	0
85–89	2
80–84	5
75–79	15
70–74	6
65–69	3
60–64	2

78.

INT.	f
175–199	4
150–174	5
125–149	7
100–124	11
75–99	12
50–74	9
25–49	7
0–24	3

79.

INT.	f
105–109	3
100–104	7
95–99	11
90–94	13
85–89	12
80–84	9
75–79	3
70–74	5

80.

INT.	f
60–62	1
57–59	3
54–56	4
51–53	5
48–50	8
45–47	10
42–44	6
39–41	7
36–38	3
33–35	2
30–32	0
27–29	1

81.

INT.	f
200–209	3
190–199	0
180–189	2
170–179	4
160–169	5
150–159	7
140–149	6
130–139	3
120–129	4
110–119	1
100–109	1

82.

INT.	f
90–99	3
80–89	6
70–79	9
60–69	5
50–59	14
40–49	4
30–39	3
20–29	1
10–19	0
0–9	2

83.

INT.	f
140–146	2
133–139	4
126–132	3
119–125	0
112–118	7
105–111	9
98–104	12
91–97	10
84–90	8
77–83	5
70–76	3
63–69	1

84.

INT.	f
36–38	1
33–35	4
30–32	0
27–29	7
24–26	9
21–23	15
18–20	8
15–17	6
12–14	3
9–11	2

85.

INT.	f
95–99	5
90–94	0
85–89	4
80–84	3
75–79	7
70–74	9
65–69	13
60–64	21
55–59	17
50–54	9
45–49	4
40–44	2
35–39	1
30–34	1

FINDING THE MEDIAN BY INSPECTION

(*a*) When the number *yet to be used* happens to equal one-half the frequency of the mid-interval, the median is the *mid-point of the mid-interval.*

ILLUSTRATIVE PROBLEM, *with solution.* —

INT.	f	
90–99	4	
80–89	8	
70–79	12	(24)
60–69	20	
50–59	10	
40–49	8	
30–39	4	
20–29	2	
	N 68	

In this problem $\frac{N}{2} = 34$, which is 10 more than the total frequency above the mid-interval; 10 is one-half of 20, the frequency of the mid-interval; so we have

$Md. = 65.0.$

$Md. = 70.0 - \frac{10}{20} \times 10 = 65.0.$

(*b*) When the number *yet to be used* when counting downward happens to equal the frequency of the mid-interval, then the median is the *bottom of the mid-interval.*

ILLUSTRATIVE PROBLEM, *with solution.* —

INT.	*f*	
75–79	3	
70–74	6	
65–69	7	(16)
60–64	8	
55–59	5	
50–54	4	
45–49	6	
40–44	5	
35–39	4	
	N 48	

In this problem $\frac{N}{2} = 24$, so the number *yet to be used* is 8. This is the total frequency of the mid-interval, so $Md. = 60.0$.
$Md. = 65.0 - \frac{8}{8} \times 5 = 60.0$.

(*c*) When the number *yet to be used* is any simple fraction of the frequency of the mid-interval, such as $\frac{1}{4}$, $\frac{1}{10}$, $\frac{1}{5}$, etc., the median may be obtained by inspection.

ILLUSTRATIVE PROBLEM, *with solution.* —

INT.	*f*	
160–179	5	
140–159	7	
120–139	10	
100–119	19	(41)
80–99	16	
60–79	17	
40–59	10	
20–39	4	
0–19	2	
	N 90	

In this problem the number yet to be used is 4, which is $\frac{1}{4}$ of 16, the frequency of the mid-interval. Hence $Md. = 95.0$.
$Md. = 100 - \frac{4}{16} \times 20 = 95.0$

ILLUSTRATIVE PROBLEM, *with solution.* —

INT.	f	
90–99	4	
80–89	7	
70–79	6	(17)
60–69	5	
50–59	5	
40–49	4	
30–39	3	
20–29	0	
10–19	3	
0–9	1	
	N 38	

In this problem the number yet to be used is 2. Hence we have

$Md. = 66.0.$

$Md. = 70.0 - \frac{2}{5} \times 10 = 66.0.$

NOTE. — In the following problems the only parts of the distribution shown are:

The mid-interval and its frequency.
The total f above the mid-interval.
The number of cases.

ILLUSTRATIVE PROBLEM, *with solution.* —

		(20)
65–69	8	
	N 48	

$Md. = 67.5$

ILLUSTRATIVE PROBLEM, *with solution.* —

		(30)
80–89	6	
	N 72	

$Md. = 80.0$

PROBLEMS TO SOLVE

Find the median by inspection:

1.

		(12)
50–59	16	
	N 40	

2.

		(10)
72–73	8	
	N 36	

3.

		(44)
40–49	12	
	N 100	

4.

	(18)
35–39	9
N	45

5.

	(19)
75–89	6
N	50

6.

	(20)
63–65	14
N	54

7.

	(20)
80–84	10
N	44

8.

	(225)
100–119	50
N	500

9.

	(30)
60–74	5
N	65

10.

	(36)
45–49	8
N	80

11.

	(24)
60–69	11
N	70

12.

	(41)
80–99	18
N	100

13.

	(27)
90–92	9
N	60

14.

	(16)
75–79	5
N	42

15.

	(30)
50–59	7
N	67

16.

	(40)
120–129	11
N	91

17.

	(40)
60–74	12
N	88

18.

	(25)
15–19	15
N	56

19.

	(45)
80–89	20
N	100

20.

	(29)
95–99	6
N	70

21.

	(40)
18–19	13
N	93

22.

	(70)
70–74	10
N	150

23.

	(34)
100–109	10
N	70

24.

	(65)
69–72	10
N	140

25.

	(18)
49–55	11
N	47

26.

	(200)
160–179	48
N	424

27.

	(44)
42–43	4
N	90

28.

	(33)
64	10
N	80

29.

	(30)
175–199	15
N	84

30.

	(70)
400–449	5
N	145

31.

	(30)
50–54	7
N	67

32.

	(24)
20–21	16
N	56

33.

	(80)
60–62	9
N	172

34.

	(37)
40–59	8
N	90

35.

	(35)
70–74	13
N	83

36.

	(38)
60–79	16
N	100

37.

	(60)
30–32	12
N	128

38.

	(45)
55–59	1
N	92

39.

	(175)
42–47	75
N	400

40.

	(71)
80–89	20
N	150

41.

	(45)
60–74	7
N	97

42.

	(50)
40–59	40
N	176

43.

	(27)
100–104	15
N	60

44.

	(36)
90–99	8
N	84

45.

	(62)
75	10
N	130

46.

	(38)
155–159	20
N	84

47.

	(50)
70–71	13
N	126

48.

	(50)
65–69	14
N	114

49.

	(48)
62	20
N	100

50.

	(125)
81–83	50
N	300

51.

	(60)
100–109	30
N	126

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CHAPTER XIII

THE MODE

Explanation. — The *mode* is the most frequent measure in a series. In the case of a simple series of measures, we find the one which occurs most often. In the following group,

92, 87, 62, 84, 87, 65, 62, 87, 92, 90, 85, 87, 90

the mode is 87, because it occurs more frequently than any other measure.

ILLUSTRATIVE PROBLEM, *with solution.* —

Find the mode:

95, 96, 87, 42, 90, 96, 42, 75, 63, 42, 54, 58, 90.

Mode = 42 (Ans.)

Not every group of measures has a single mode. There may be two, three, or more modes. If a series contains two modes, it is said to be *bimodal;* if it contains three or more modes, it is called *trimodal*, or *multimodal.*

ILLUSTRATIVE PROBLEMS, *with solutions.* —

(*a*) Find the modes:

16, 15, 18, 17, 14, 15, 13, 17, 15, 19, 20, 15, 17, 13, 17.

Modes = 15, 17. (Ans.)

(*b*) Find the modes:

30, 27, 42, 25, 38, 35, 26, 25, 35, 40, 42, 38, 31, 36, 38, 39, 42, 35, 34, 37, 33. Modes = 35, 38, 42. (Ans.)

PROBLEMS TO SOLVE

Find the mode, or modes, in each group:

1. 62, 83, 75, 96, 62, 66, 75, 68, 70, 75, 74, 75, 76.

2. 35, 55, 45, 65, 75, 45, 65, 35, 45, 85, 40, 45, 33, 38, 45.

3. 49, 56, 50, 53, 46, 50, 47, 56, 48, 45, 44, 50, 49, 48, 57, 58, 40, 37, 48, 35, 59, 48, 41, 40, 51, 50, 65, 62.

4. 75, 81, 74, 72, 90, 85, 81, 70, 90, 83, 74, 83, 81, 70, 90, 73, 84, 90, 73, 75, 81, 85, 81, 72, 90, 70, 83, 96, 75.

5. 15, 13, 19, 15, 10, 15, 12, 15, 19, 15, 18, 18, 14, 14, 13, 14, 10, 16, 13, 16, 10, 19, 16, 14, 16, 12, 16, 18, 16, 12.

6. 5, 4, 10, 5, 2, 8, 3, 10, 7, 8, 5, 3, 16, 8, 6, 5, 15, 8, 7, 14, 10, 4, 2, 15, 10, 6, 4, 3, 13, 15, 8, 6, 2, 10, 7, 13, 14.

7. 135, 130, 125, 140, 150, 125, 135, 140, 125, 150, 115, 160, 125, 160, 130, 125, 150, 115, 125, 115, 135.

8. 20, 40, 30, 50, 50, 30, 50, 20, 90, 30, 70, 20, 30, 70, 70, 30, 40, 60, 80, 30, 60, 80, 90, 30, 20, 60, 30, 20, 90, 30, 40.

9. 86.3, 85.7, 87.5, 86.0, 86.3, 88.2, 87.5, 84.5, 88.4, 87.5, 88.4, 85.7, 84.5, 86.0, 88.2, 87.5, 87.3, 84.5, 88.4, 87.5, 86.0, 88.2, 86.3.

10. 75, 70, 85, 50, 80, 65, 75, 90, 60, 50, 90, 65, 95, 50, 95, 90, 65, 85, 75, 70, 60, 85, 50, 80, 75, 90, 50, 60, 75, 90, 95.

11. 50, 65, 50, 70, 30, 40, 40, 60, 55, 30, 10, 30, 20, 60, 20, 90, 90, 75, 60, 70, 90, 80, 90, 80, 30, 10, 30, 60, 95, 90, 20, 60, 10, 60, 80, 90, 45, 90, 85, 30, 20, 30, 70, 60, 50, 55, 40.

12. 104, 105, 102, 104, 105, 103, 106, 100, 109, 104, 102, 109, 100, 102, 100, 102, 110, 115, 100, 106, 100, 106, 100, 106, 100, 103, 109, 110.

The modal interval. — When we have a frequency distribution we cannot determine which measure occurs the greatest number of times, because the individual cases have lost their identity. We can, however, locate the *modal interval*, which is the *interval having the largest frequency.*

ILLUSTRATIVE PROBLEM, *with solution.* —

Find the modal interval:

INT.	*f*	
90–99	3	
80–89	8	
70–79	15	Modal interval, 70–79. (Ans.)
60–69	12	
50–59	10	
40–49	9	
30–39	7	
20–29	0	
10–19	2	
	N 66	

It will be observed in the foregoing distribution that the frequencies increase up to the modal interval, and then gradually decrease. It is the nature of most distributions to have some outstanding interval, which is usually (but not always) near the center. This modal interval may or may not be the mid-interval, and the two should not be confused.

A distribution may be *bimodal*, or *multimodal*, if it has two or more outstanding points. The frequency of the modal intervals in such a distribution need not be exactly equal, for these points are called modes if they are noticeably outstanding. For example:

INT.	f
75–79	7
70–74	9
65–69	12
60–64	8
55–59	4
50–54	6
45–49	15
40–44	7
35–39	2
N	70

This is a *bimodal* distribution, the modal intervals being 65–69 and 45–49.

INT.	f
150–159	2
140–149	3
130–139	8
120–129	16
110–119	7
100–109	4
90–99	15
80–89	9
70–79	2
60–69	13
50–59	6
N	85

This is a *trimodal* distribution, having three modes. The modal intervals are 120–129, 90–99, and 60–69.

PROBLEMS TO SOLVE

Find the modal intervals:

1.

INT.	f
75–79	3
70–74	4
65–69	7
60–64	15
55–59	8
50–54	4
45–49	0
40–44	2

2.

INT.	f
90–99	2
80–89	0
70–79	5
60–69	10
50–59	8
40–49	7
30–39	5
20–29	3
10–19	2

3.

INT.	f
180–189	12
170–179	15
160–169	27
150–159	36
140–149	42
130–139	35
120–129	28
110–119	13
100–109	10

4.

INT.	*f*
95–99	2
90–94	0
85–89	5
80–84	12
75–79	9
70–74	7
65–69	10
60–64	15
55–59	7
50–54	3
45–49	2

5.

INT.	*f*
48–49	1
46–47	0
44–45	1
42–43	2
40–41	3
38–39	7
36–37	4
34–35	2
32–33	5
30–31	8
28–29	3

6.

INT.	*f*
90–92	5
87–89	7
84–86	9
81–83	15
78–80	21
75–77	18
72–74	12
69–71	14
66–68	27
63–65	20
60–62	14
57–59	6

7.

INT.	*f*
175–179	2
170–174	12
165–169	9
160–164	15
155–159	10
150–154	6
145–149	9
140–144	14
135–139	8
130–134	6
125–129	2

8.

INT.	*f*
90–99	3
80–89	5
70–79	7
60–69	12
50–59	15
40–49	24
30–39	27
20–29	20
10–19	10

9.

INT.	*f*
65–69	25
60–64	15
55–59	14
50–54	12
45–49	10
40–44	10
35–39	7
30–34	5

Complete the intervals and find the modal intervals:

10.

INT.	f
60–62	3
	2
	4
	7
	12
	10
	8
	2
	0
	1

11.

INT.	f
145–149	4
	3
	8
	10
	16
	25
	20
	15
	10
	4

12.

INT.	f
200–209	2
	0
	0
	3
	7
	9
	18
	27
	32
	20

13.

INT.	f
95–99	1
	1
	0
	2
	6
	9
	7
	3
	3
	2

14.

INT.	f
78–79	5
	6
	8
	10
	7
	3
	2
	4
	9
	7
	3

15.

INT.	f
93	2
	0
	2
	4
	0
	7
	12
	6
	8
	15
	2

16.		17.		18.	
INT.	*f*	INT.	*f*	INT.	*f*
240–259	6	36–38	4	100–109	15
	8		15		23
	12		10		32
	27		7		48
	35		7		56
	30		3		50
	25		0		40
	15		4		60
	10		2		32
	8		2		10

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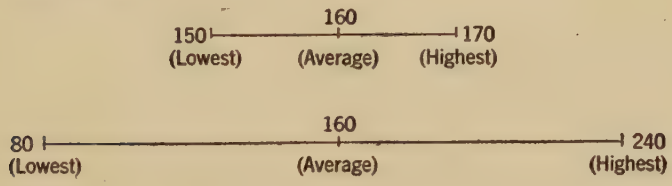
PART III

MEASURES OF VARIABILITY, OR DEVIATION

CHAPTER XIV

MEANING OF VARIABILITY

Explanation. — It will be recalled that a central tendency is used to *represent* a group of measures, but that it can do so only in a general way. To say that the average weight of a group of men is 160 pounds tells us nothing about the extent to which individual men in the group differ from 160 pounds. In one group of men averaging 160 pounds in weight there might be a man weighing only 80 pounds and another weighing 240. On the other hand the extreme weights might be only 150 and 170, the average in either case being the same. These two examples may be illustrated thus:



While it is correct to say that these two groups are equal in weight, from the standpoint of the average, it would be misleading to refer to them as being the same, if we consider at all the weights of the individuals concerned. In this case the characteristic difference lies in the range of the measures. The greater the range, the greater the differences among the individual measures. These individual differences give a group of measures a quality which we call *variability*, or *dis-*

persion. Measures of variability indicate the spread of the individual values on either side of their central tendency.

The range as a measure of variability. — In the foregoing illustration the greater range of the lower group shows that it is more variable than the upper group. The range of the upper group is only one-eighth as great. So the range is a measure of variability, and is sometimes so used. It is easy to see the difference between the two following groups:

A. Median 85.3, range 50–127.
B. Median 85.3, range 80–95.

The difference could be explained by saying that while groups *A* and *B* have the same central tendency, group *A* is the more *variable.*

While the range may be considered a measure of variability for such general descriptions, it is not a satisfactory measure for most statistical purposes. There are two reasons for this: first, because we often have measures arranged in a frequency distribution, from which the exact range cannot be determined; second, because the range is indicated by only two individual measures — the highest and the lowest — and these may not, and often do not, sufficiently represent the other measures to justify their use as indications of the group.

This point may be illustrated as follows: Suppose a class of pupils have intelligence quotients as follows:

95, 97, 99, 99, 99, 100, 100, 100, 101, 102, 103, 103, 104, 105. The range is 95–105, with a central tendency (mid-score) of 100. Now suppose the lowest pupil, testing 95, should leave school and another pupil,

testing 80, should enter. The range would then become 80–105, without affecting the mid-score at all. That is, the significant features of the group are not affected by this change, and yet a range of 80–105 suggests, at first sight, a significantly greater variability than does 95–105.

Better measures than the range. — The inadequacy of the range as a measure of variability makes necessary the use of other measures. In this book we shall study three: (1) the average deviation, (2) the standard deviation, and (3) the quartile deviation. Methods are shown for computing the average deviation and the standard deviation from both simple series and frequency distribution; the quartile deviation is shown for frequency distribution only.

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CHAPTER XV

THE AVERAGE DEVIATION

Explanation. — The *average deviation* (*A.D.*) is the common average of individual deviations from some central tendency, without regard to the sign (plus or minus) of the differences. The formula (when the measures are in a simple series) is

$$A.D. = \frac{\Sigma d}{N}.$$

ILLUSTRATIVE PROBLEM. —

Find *A.D.* from the common average:

67	*Solution:*		
75	m	d	
69	67	3	
65	75	5	$N = 10$
72	69	1	Each deviation is
72	65	5	measured from 70,
68	72	2	the average.
70	72	2	$A.D. = \frac{\Sigma d}{N}$
71	68	2	
71	70	0	$A.D. = \frac{22}{10}$
	71	1	$A.D. = 2.2$ (Ans.)
	71	1	
	10)700	10)22	
	70 *M*	2.2 *A.D.*	

ILLUSTRATIVE PROBLEM. —

Find *A.D.* from the common average:

Solution:

	m	*d*
8	8	2.5
5	5	.5
9	9	3.5
0	0	5.5
7	7	1.5
6	6	.5
3	3	2.5
8	8	2.5
4	4	1.5
7	7	1.5
5	5	.5
4	4	1.5
6	6	.5
5	5	.5
	77	25.0 (Σd)

$N = 14$

$M = \frac{77}{14}$ or 5.5

$A.D. = \frac{\Sigma d}{N}$

$A.D. = \frac{25}{14}$

$A.D. = 1.78$ (Ans.)

ILLUSTRATIVE PROBLEM. —

Find *A.D.* from the common average:

Solution:

	m	*d*
82	82	3.3
70	70	8.7
65	65	13.7
62	62	16.7
83	83	4.3
80	80	1.3
80	80	1.3
74	74	4.7
92	92	13.3
99	99	20.3
	787	87.6 (Σd)

$N = 10$

$M = 78.7$

$A.D. = \frac{\Sigma d}{N}$

$A.D. = \frac{87.6}{10}$

$A.D. = 8.76$ (Ans.)

ILLUSTRATIVE PROBLEM. —

Find *A.D.* from the mid-score:

		Solution: *m*	*d*
94		99	7
88		98	6
98		98	6
91		96	4
80		95	3
99		94	2
80	mid-score	(92)	0
77		91	1
95		88	4
98		80	12
92		80	12
96		78	14
78		77	15
			86 (Σd)

(Note: Numbers are arranged in order for convenience in finding the mid-score.)

$N = 13$

Mid-score = 92

$A.D. = \frac{\Sigma d}{N}$

$A.D. = \frac{86}{13}$

$A.D. = 6.6$ (Ans.)

ILLUSTRATIVE PROBLEM. —

Find *A.D.* from the mode:

	Solution: *m*	*d*
75	75	8
83	(83)	0
76	76	7
80	80	3
83	(83)	0
82	82	1
81	81	2
83	(83)	0
79	79	4
80	80	3
83	(83)	0
85	85	2
		30 (Σd)

$N = 12$

Mode = 83

$A.D. = \frac{\Sigma d}{N}$

$A.D. = \frac{30}{12}$

$A.D. = 2.5$ (Ans.)

Outline of procedure, for a simple series of scores. —

(*a*) Place measures in a vertical column. (It is not always necessary to have them arranged in order, but it will be a convenience to do so.)

(*b*) Find the central tendency from which the *A.D.* is to be found.

(*c*) Opposite each measure write its deviation; that is, its difference from the central tendency.

(*d*) Find the sum of the deviations.

(*e*) Divide by N.

Formula: $A.D. = \frac{\Sigma d}{N}$.

PROBLEMS TO SOLVE

Find A.D. from the common average:

1. 80, 70, 93, 87, 80, 75, 70, 75, 80, 70.
2. 9, 8, 0, 10, 7, 4, 9, 0, 0, 9, 0, 7, 8, 9, 0, 9, 8, 2.
3. 69, 60, 56, 70, 75, 66, 70, 62, 58, 65, 73, 68, 58.
4. 85, 60, 40, 50, 95, 80, 90, 65, 75, 100, 30, 55, 70, 45, 35.
5. 77, 85, 95, 87, 75, 83, 91, 93, 81, 89, 79.
6. 62, 57, 65, 60, 58, 50, 61, 64, 68, 54, 62, 59.
7. 95, 92, 94, 90, 92, 95, 85, 91, 93, 94, 90, 85, 90, 94, 85.
8. 83, 80, 80, 77, 80, 77, 80, 83, 77, 80, 77, 83, 77, 80, 83, 80, 83, 77, 83, 80.
9. 71, 65, 69, 67, 75, 62, 66, 68, 70, 74, 64, 63, 73, 60, 72, 61, 59.
10. 115, 120, 100, 100, 116, 117, 110, 113, 113, 118, 110, 118, 100, 115, 100.

Find A.D. from the mid-score:

11. 72, 68, 65, 75, 74, 70, 60, 56, 58, 63, 69.
12. 94, 84, 90, 92, 81, 89, 87, 98, 94, 95, 86, 97, 82.
13. 90, 88, 82, 94, 80, 84, 85, 88, 91, 85, 88, 92.
14. 68, 58, 69, 67, 70, 61, 60, 68, 62, 66, 60, 64, 69.
15. 90, 100, 80, 75, 85, 90, 95, 100, 75, 85, 90, 95, 75, 95.
16. 36, 27, 21, 35, 36, 31, 26, 23, 27, 25, 22, 30, 35, 30, 32.
17. 130, 120, 115, 136, 130, 124, 120, 135, 124, 127, 128, 115, 135, 115, 138.
18. 80, 70, 76, 75, 84, 88, 75, 72, 69, 65, 78, 80, 70, 67, 62, 75, 86.
19. 40, 32, 30, 28, 43, 45, 32, 31, 27, 25, 45, 40, 36, 33, 25, 35, 42, 38, 44, 30.
20. 10, 7, 9, 4, 4, 10, 12, 7, 9, 0, 3, 8, 9, 4, 0, 8, 10, 12, 13.
21. 95, 85, 80, 87, 88, 92, 83, 80, 87, 100, 90, 100, 85, 92, 95, 80, 100, 85.
22. 7, 2, 8, 3, 7, 4, 3, 9, 1, 9, 0, 5, 6, 7, 9, 6, 5, 2, 9, 8, 6.
23. 67, 61, 68, 70, 73, 69, 70, 72, 67, 65, 64, 69, 70, 68, 68, 65, 62, 69, 72, 69, 72, 65.
24. 36, 34, 30, 42, 38.5, 33, 30, 25, 32.5, 38, 40, 43, 35, 30, 35.5, 39, 37.
25. 106, 110, 100, 96, 100, 104, 106, 104, 106, 98, 102, 104, 100, 104, 108, 102, 100, 95, 103, 104, 106, 108, 109, 86, 110, 95.
26. 14, 9, 6, 5, 12, 7, 15, 10, 5, 14, 6, 5, 15, 10, 13, 7, 15, 16, 4, 13, 5, 12.
27. 215, 200, 210, 198, 210, 218, 220, 200, 190, 217, 195, 188, 180, 210, 220, 208, 204, 188, 206, 215, 217, 185, 182.
28. 24, 38, 23, 15, 12, 10, 25, 27, 35, 15, 26, 37, 16, 12, 10, 16, 18, 17, 23, 20, 30, 18, 14, 12, 10.
29. 44, 32, 40, 47, 30, 20, 19, 48, 47, 52, 35, 40, 25, 47, 45, 30, 18, 38, 40, 28, 20, 48, 20, 45, 50, 18.
30. 14, 11, 10, 16, 9, 18, 10.5, 8.5, 13.5, 19, 14, 12, 13.5, 10, 7.5, 14, 7.5, 12, 15.5, 19, 9.5, 8, 17.5, 12, 19.
31. 94, 95, 96, 95.5, 93, 93.7, 91, 90, 92, 90.5, 91.5, 93, 94.3, 96.5, 92, 90, 91, 94, 90, 93, 85, 93.5, 96.2, 90.
32. 37.5, 45.3, 35, 32.5, 35.2, 41.6, 46, 36, 34.2, 36.3, 39, 40.5, 35.8, 34.7, 34.5, 38, 43, 34, 31.5, 44.7, 36.2, 35, 32, 33, 32, 44.3.

Find A.D. from the mode:

33. 64, 60, 74, 60, 65, 60, 70, 75, 60, 74, 60, 64.
34. 65, 80, 85, 70, 60, 80, 85, 90, 95, 85, 75, 85, 90, 70, 85, 75, 65.
35. 14, 5, 8, 15, 8, 5, 15, 12, 15, 14, 10, 5, 15, 13, 15, 10.
36. 74, 60, 55, 50, 45, 55, 70, 55, 65, 75, 55, 50, 60, 55, 65, 55, 72, 74, 60, 50.
37. 10.5, 10.0, 8.0, 9.5, 10.5, 12.0, 10.0, 10.5, 11.5, 9.0, 5.0, 9.5, 10.0, 11.0, 10.5, 10.5, 8.0, 5.0.
38. 84, 80, 79, 80, 80, 78, 75, 80, 82, 80.
39. 62, 60, 58, 60, 62, 58, 60, 58, 62, 60.
40. 100, 98, 97, 99, 98, 94, 98, 90, 98, 90, 98, 98.
41. 65, 70, 60, 65, 60, 70, 65, 70, 60, 75.
42. 78, 87, 86, 85, 88, 86, 87, 88, 86, 85, 88, 87, 88, 85.
43. 63, 60, 61, 64, 67, 63, 64, 60, 61, 63, 64, 60, 65, 63, 65, 63.
44. 40, 44, 38, 42, 43, 42, 37, 40, 38, 42, 43, 44, 36, 42, 40, 42, 44, 45.

THE AVERAGE DEVIATION FROM A FREQUENCY DISTRIBUTION

When measures are arranged in intervals, the *A.D.* may be calculated from any central tendency which a frequency distribution is capable of yielding. For example, it may be found from the estimated mean, the arithmetic mean, or the median. The procedure is as follows:

(*a*) Find the central tendency desired.
(*b*) Indicate, for each interval, its mid-point.
(*c*) Find the deviation (without regard to sign) of each mid-point from the central tendency used.
(*d*) Multiply each of these deviations by its corresponding frequency.
(*e*) Divide the sum of these frequency deviations by *N*.

The formula is

$$A.D. = \frac{\Sigma fd}{N}.$$

ILLUSTRATIVE PROBLEM, *with solution.* —
(To find *A.D.* from the estimated mean)

INT.	f	MID-POINT	d	fd
60–69	3	65	30	90
50–59	5	55	20	100
40–49	6	45	10	60
30–39	7	35	0	0
20–29	10	25	10	100
10–19	4	15	20	80
0–9	2	5	30	60
	N 37			490

Est. $M = 35.0$

$$A.D. = \frac{\Sigma fd}{N}$$

$$A.D. = \tfrac{490}{37}$$

$A.D. = 13.2$ (Ans.)

ILLUSTRATIVE PROBLEM, *with solution.* —
(To find *A.D.* from the arithmetic mean)

	TO FIND ARITH. MEAN				TO FIND A.D.		
INT.	f	d	fd		MID-POINT	d	fd
90–99	1	+4	+4		95.0	44.6	44.6
80–89	3	+3	+9		85.0	34.6	103.8
70–79	5	+2	+10		75.0	24.6	123.0
60–69	6	+1	+6	(+29)	65.0	14.6	87.6
50–59	11	0	0		55.0	4.6	50.6
40–49	10	−1	−10		45.0	5.4	54.0
30–39	6	−2	−12		35.0	15.4	92.4
20–29	4	−3	−12		25.0	25.4	101.6
10–19	2	−4	−8		15.0	35.4	70.8
0–9	2	−5	−10	(−52)	5.0	45.4	90.8
	N 50		−23				819.2
			Σfd				

$$M = \text{Est. } M \pm \frac{\Sigma fd}{N} \times \text{int.}$$

$$M = 55.0 - \tfrac{23}{50} \times 10$$

$$M = 55.0 - \tfrac{230}{50}$$

$$M = 55.0 - 4.6$$

$$M = 50.4$$

$$M = 50.4$$

$$A.D. = \frac{\Sigma fd}{N}$$

$$A.D. = \frac{819.2}{50}$$

$A.D. = 16.3$ (Ans.)

ILLUSTRATIVE PROBLEM, *with solution.* —

(To find *A.D.* from the median)

INT.	*f*	MID-POINT	*d*	*fd*
80–84	3	82.5	20	60
75–79	5	77.5	15	75
70–74	8	72.5	10	80
65–69	10	67.5	5	50
60–64	8	62.5	0	0
55–59	18	57.5	5	90
50–54	5	52.5	10	50
45–49	0	47.5	15	0
40–44	2	42.5	20	40
35–39	1	37.5	25	25
N	60			470
				Σfd

$$\frac{N}{2} = 30$$

$$Md. = 65 - \tfrac{4}{8} \times 5 = 62.5$$

$$A.D. = \frac{\Sigma fd}{N}$$

$$A.D. = \tfrac{470}{60}$$

$$A.D. = 7.3 \text{ (Ans.)}$$

PROBLEMS TO SOLVE

Find A.D. from the estimated mean:

1.

INT.	*f*
90–99	2
80–89	3
70–79	7
60–69	9
50–59	12
40–49	8
30–39	4
20–29	3
10–19	2

2.

INT.	*f*
95–99	2
90–94	2
85–89	3
80–84	8
75–79	12
70–74	14
65–69	8
60–64	6
55–59	4
50–54	1

3.

INT.	*f*
160–179	1
140–159	1
120–139	3
100–119	6
80–99	8
60–79	10
40–59	10
20–39	9
0–19	2

4.

INT.	f
140–149	3
130–139	4
120–129	6
110–119	8
100–109	10
90–99	15
80–89	12
70–79	5
60–69	2

5.

INT.	f
130–134	4
125–129	5
120–124	7
115–119	8
110–114	12
105–109	14
100–104	6
95–99	5
90–94	3
85–89	1

6.

INT.	f
88–89	1
86–87	2
84–85	4
82–83	7
80–81	8
78–79	8
76–77	13
74–75	12
72–73	8
70–71	8
68–69	4

7.

INT.	f
90–92	2
87–89	3
84–86	5
81–83	8
78–80	10
75–77	15
72–74	10
69–71	8
66–68	4
63–65	2
60–62	2

8.

INT.	f
180–199	4
160–179	6
140–159	8
120–139	14
100–119	27
80–99	13
60–79	15
40–59	7
20–39	3
0–19	2

9.

INT.	f
110–119	5
100–109	6
90–99	8
80–89	10
70–79	15
60–69	24
50–59	18
40–49	9
30–39	9
20–29	0
10–19	4

10.

INT.	*f*
65–69	4
60–64	5
55–59	0
50–54	8
45–49	12
40–44	10
35–39	8
30–34	8
25–29	7
20–24	3

11.

INT.	*f*
80–81	3
78–79	0
76–77	0
74–75	7
72–73	10
70–71	15
68–69	24
66–67	10
64–65	8
62–63	6
60–61	4

12.

INT.	*f*
67	4
66	4
65	0
64	6
63	8
62	13
61	7
60	12
59	10
58	8
57	2
56	1

13.

INT.	*f*
90–92	5
87–89	5
84–86	4
81–83	8
78–80	5
75–77	6
72–74	7
69–71	0
66–68	0
63–65	3
60–62	2

14.

INT.	*f*
135–149	3
120–134	8
105–119	12
90–104	15
75–89	12
60–74	10
45–59	7
30–44	0
15–29	8
0–14	2

15.

INT.	*f*
140–144	7
135–139	3
130–134	4
125–129	5
120–124	8
115–119	15
110–114	14
105–109	7
100–104	5

16.

INT.	f
90–95	3
84–89	3
78–83	4
72–77	7
66–71	9
60–65	15
54–59	20
48–53	14
42–47	10
36–41	10
30–35	8
24–29	0
18–23	7
12–17	4

17.

INT.	f
200–204	3
195–199	4
190–194	5
185–189	5
180–184	7
175–179	8
170–174	12
165–169	10
160–164	10
155–159	0
150–154	8
145–149	3
140–144	3
135–139	1

18.

INT.	f
180–189	2
170–179	2
160–169	4
150–159	5
140–149	7
130–139	9
120–129	14
110–119	8
100–109	3
90–99	10
80–89	12
70–79	7
60–69	3

Find A.D. from the arithmetic mean:

19.

INT.	f
95–99	3
90–94	5
85–89	8
80–84	12
75–79	15
70–74	20
65–69	17
60–64	10
55–59	5
50–54	3
45–49	2

20.

INT.	f
95–99	5
90–94	25
85–89	25
80–84	27
75–79	35
70–74	42
65–69	65
60–64	38
55–59	20
50–54	8

21.

INT.	f
90–99	1
80–89	3
70–79	5
60–69	6
50–59	11
40–49	10
30–39	6
20–29	4
10–19	2
0–9	2

22.

INT.	*f*
200–209	2
190–199	0
180–189	4
170–179	6
160–169	4
150–159	3
140–149	2
130–139	1
120–129	0
110–119	2
100–109	1

23.

INT.	*f*
85–89	2
80–84	3
75–79	1
70–74	6
65–69	10
60–64	16
55–59	25
50–54	15
45–49	9
40–44	5
35–39	7
30–34	1

24.

INT.	*f*
80	2
79	1
78	3
77	0
76	8
75	12
74	10
73	6
72	2
71	3
70	2
69	1

25.

INT.	*f*
120–129	12
110–119	6
100–109	6
90–99	15
80–89	23
70–79	69
60–69	31
50–59	7
40–49	15
30–39	4
20–29	3
10–19	9

26.

INT.	*f*
55–59	2
50–54	6
45–49	12
40–44	7
35–39	13
30–34	14
25–29	11
20–24	10
15–19	8
10–14	8
5–9	5
0–4	4

27.

INT.	*f*
70	2
69	1
68	3
67	0
66	8
65	12
64	10
63	6
62	2
61	3
60	2
59	1

Find A.D. from the median:

28.

INT.	f
95–99	3
90–94	5
85–89	10
80–84	13
75–79	8
70–74	7
65–69	6
60–64	5
55–59	5
50–54	4
45–49	3
40–44	1

29.

INT.	f
90–99	4
80–89	5
70–79	7
60–69	8
50–59	6
40–49	7
30–39	8
20–29	6
10–19	5
0–9	4

30.

INT.	f
72–73	1
70–71	2
68–69	3
66–67	7
64–65	7
62–63	10
60–61	10
58–59	4
56–57	3
54–55	0
52–53	0
50–51	3

31.

INT.	f
60–62	1
57–59	2
54–56	2
51–53	3
48–50	5
45–47	4
42–44	4
39–41	9
36–38	12
33–35	4
30–32	2

32.

INT.	f
75–79	2
70–74	3
65–69	4
60–64	5
55–59	8
50–54	8
45–49	10
40–44	9
35–39	8
30–34	5

33.

INT.	f
90–94	3
85–89	7
80–84	8
75–79	7
70–74	10
65–69	8
60–64	6
55–59	7
50–54	4

34.

INT.	*f*
90–99	1
80–89	3
70–79	2
60–69	4
50–59	5
40–49	10
30–39	8
20–29	7
10–19	2
0–9	2

35.

INT.	*f*
220–239	3
200–219	4
180–199	3
160–179	8
140–159	4
120–139	6
100–119	6
80–99	2
60–79	0
40–59	2

36.

INT.	*f*
80	3
79	4
78	7
77	6
76	5
75	2
74	0
73	3
72	4
71	4
70	2
69	2

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CHAPTER XVI

THE STANDARD DEVIATION

Explanation. — The *standard deviation* is a widely used measure of variability, especially in technical statistical procedure, and is often referred to in reports of investigations. For the analysis of a large group of cases it is of greater significance than the preceding measures. The standard deviation differs from the average deviation in that it is based on the square root of the sum of the squares of the individual deviations from the mean.* This sort of treatment of the data brings about a refined, accurate, and statistically important result.

The standard deviation may be calculated from either a simple series of measures or a frequency distribution. It is represented by the small Greek letter *sigma*, written σ.

THE STANDARD DEVIATION FROM A SIMPLE SERIES

To obtain σ from a simple series, we have but to find the individual deviations from the mean, as in the case of *A.D.*; then to square these deviations, find the sum of the squares, divide by N, and extract the square root. The formula is

$$\sigma = \sqrt{\frac{\Sigma d^2}{N}}$$

* The standard deviation is always based on the mean unless specifically stated otherwise.

wherein Σd^2 is the sum of the squared deviations, and N is the number of cases.

Outline of procedure. — Simply list the measures in a vertical column (they need not be arranged in order, although it may be a convenience to do so). The steps are as follows:

(*a*) Find the sum of the measures.
(*b*) Divide by N to find the mean (common average).
(*c*) Find the deviation of each measure from the mean, and list in the column headed "*d*".
(*d*) Find the square of each deviation. List in the column headed "d^2".
(*e*) Find the sum of the d^2 column (Σd^2).
(*f*) Divide Σd^2 by N.
(*g*) Find the square root. The answer is the standard deviation (σ).

ILLUSTRATIVE PROBLEM. — Find the standard deviation of the following measures:

75, 77, 64, 72, 80, 68, 70, 68, 60, 76.

Solution:

MEASURES	d	d^2
75	4	16
77	6	36
64	7	49
72	1	1
80	9	81
68	3	9
70	1	1
68	3	9
60	11	121
76	5	25
10)710		348 (Σd^2)
71.0 (Mean)		

$$\sigma = \sqrt{\frac{\Sigma d^2}{N}}$$

$$\sigma = \sqrt{\frac{348}{10}}$$

$$\sigma = \sqrt{34.8}$$

$$\sigma = 5.9 \text{ (Ans.)}$$

ILLUSTRATIVE PROBLEM. — Find the standard deviation of the following numbers:

15, 12, 10, 15, 16, 10, 15, 14, 16, 13.

Solution:

MEASURES	d	d^2
15	1.4	1.96
12	1.6	2.56
10	3.6	12.96
15	1.4	1.96
16	2.4	5.76
10	3.6	12.96
15	1.4	1.96
14	0.4	.16
16	2.4	5.76
13	0.6	.36
10)136		46.40 (Σd^2)
13.6 (Mean)		

$$\sigma = \sqrt{\frac{\Sigma d^2}{N}}$$

$$\sigma = \sqrt{\frac{46.4}{10}}$$

$$\sigma = \sqrt{4.64}$$

$$\sigma = 2.15 \text{ (Ans.)}$$

(NOTE: Use square-root tables, page 214.)

PROBLEMS TO SOLVE

Find the standard deviation (σ):

1. 84, 78, 70, 84, 65, 64, 80, 85, 80, 74.
2. 60, 64, 59, 60, 64, 67, 59, 60, 64, 59, 65, 63.
3. 70, 65, 62, 69, 71, 67, 69, 65, 64, 70, 67, 65.
4. 80, 79, 78, 77, 75, 75, 75, 74, 73, 72, 72, 70.
5. 94, 97, 90, 95, 96, 92, 91, 94, 90, 90, 91, 97, 89, 94, 95.
6. 8, 9, 7, 10, 10, 7, 4, 8, 10, 8, 7, 10, 5, 9, 8.
7. 64, 68, 58, 50, 71, 62, 61, 53, 65, 67, 70, 55.
8. 80, 70, 66, 96, 75, 74, 100, 62, 92, 79, 95, 89, 66, 96, 85, 86, 66.
9. 118, 95, 110, 100, 119, 120, 113, 107, 100, 115, 114, 119, 105, 110, 98, 90, 125, 95.
10. 140, 144, 132, 124, 120, 124, 144, 140, 142, 130, 126, 124, 132, 137, 133, 132, 127, 125.

11. 58, 52, 67, 71, 68, 70, 70, 65, 60, 57, 54, 57, 65, 62, 58, 52, 64, 65.
12. 49, 45, 48, 41, 38, 50, 44, 45, 32, 33, 39, 50, 47, 24, 32, 39, 40, 42.
13. 260, 220, 161, 216, 228, 135, 244, 188, 220, 236, 232, 240.
14. 52, 46, 47, 54, 59, 56, 52, 51, 56, 58, 46, 43, 50, 53, 56, 60, 44, 44.
15. 10.6, 15.0, 6.6, 10.7, 7.3, 12.5, 5.3, 4.1, 9.4, 9.4, 12.8, 16.3, 7.5, 4.2, 7.8.
16. 70, 54, 73, 57, 65, 75, 57, 65, 47, 74, 69, 58, 55, 65, 70, 49, 67, 43, 35, 72.
17. 26, 29, 16, 22, 20, 17, 22, 24, 27, 22, 18, 12, 14, 19, 13, 10, 19, 16, 12, 22.
18. 244, 247, 237, 243, 243, 249, 235, 233, 247, 243, 245, 244, 240, 238, 239, 247, 250, 243, 235, 238.

THE STANDARD DEVIATION FROM A FREQUENCY DISTRIBUTION

Outline of procedure. — The steps in computing the standard deviation (σ) from a frequency distribution are as follows:

(*a*) Find N.
(*b*) Find the mid-interval.
(*c*) Indicate the deviation of each interval as in finding the arithmetic mean.
(*d*) Find Σfd.
(*e*) Multiply each fd by its corresponding d, making another column, headed fd^2. (These will all be positive numbers, inasmuch as they result invariably from the multiplication of like signs.)
(*f*) Find Σfd^2 (the sum of the entire column).

(*g*) Divide Σfd^2 by N, and from the result subtract the *square* of $\frac{\Sigma fd}{N}$; that is, $\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2$.

(*h*) Extract the square root of the remainder.

(*i*) Multiply by the size of the interval.

The formula is:

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \times \text{size of int.}$$

ILLUSTRATIVE PROBLEM, *with solution.* —

Find standard deviation:

INT.	f
95–99	1
90–94	2
85–89	5
80–84	5
75–79	4
70–74	3
65–69	2
60–64	2
55–59	1

Solution:

INT.	f	d	fd	fd^2
95–99	1	+3	+3	9
90–94	2	+2	+4	8
85–89	5	+1	+5	5
80–84	5	0	0	0
75–79	4	−1	−4	4
70–74	3	−2	−6	12
65–69	2	−3	−6	18
60–64	2	−4	−8	32
55–59	1	−5	−5	25
	N 25		−17	113 = Σfd^2
			Σfd	

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \times \text{int.}$$

$$\sigma = \sqrt{\tfrac{113}{25} - (\tfrac{17}{25})^2} \times 5$$

$$\sigma = \sqrt{4.52 - .46} \times 5$$

$$\sigma = \sqrt{4.06} \times 5$$

$$\sigma = 2.0 \times 5$$

$$\sigma = 10.0 \text{ (Ans.)}$$

ILLUSTRATIVE PROBLEM, *with solution.* —

Find standard deviation:

INT.	f
90–99	2
80–89	3
70–79	5
60–69	8
50–59	12
40–49	9
30–39	5
20–29	4
10–19	2

Solution:

INT.	f	d	fd	fd^2
90–99	2	+4	+8	32
80–89	3	+3	+9	27
70–79	5	+2	+10	20
60–69	8	+1	+8	8
50–59	12	0	0	0
40–49	9	−1	− 9	9
30–39	5	−2	−10	20
20–29	4	−3	−12	36
10–19	2	−4	− 8	32
N	50		− 4 $= \Sigma fd$	184 $= \Sigma fd^2$

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \times \text{int.}$$

$$\sigma = \sqrt{\tfrac{184}{50} - \left(\tfrac{4}{50}\right)^2} \times 10$$

$$\sigma = \sqrt{3.68 - .0064} \times 10$$

$$\sigma = \sqrt{3.67} \times 10$$

$$\sigma = 1.91 \times 10$$

$$\sigma = 19.1 \text{ (Ans.)}$$

ILLUSTRATIVE PROBLEM, *with solution.* —

Find standard deviation:

INT.	f
75–79	3
70–74	4
65–69	0
60–64	5
55–59	7
50–54	15
45–49	12
40–44	10
35–39	8
30–34	6

Solution:

INT.	f	d	fd	fd^2
75–79	3	+6	+18	108
70–74	4	+5	+20	100
65–69	0	+4	0	0
60–64	5	+3	+15	45
55–59	7	+2	+14	28
50–54	15	+1	+15	15
45–49	12	0	0	0
40–44	10	−1	−10	10
35–39	8	−2	−16	32
30–34	6	−3	−18	54
	N 70		38 Σfd	392 $= \Sigma fd^2$

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \times \text{int.}$$

$$\sigma = \sqrt{\tfrac{392}{70} - \left(\tfrac{38}{70}\right)^2} \times 5$$

$$\sigma = \sqrt{5.60 - .29} \times 5$$

$$\sigma = \sqrt{5.31} \times 5$$

$$\sigma = 2.30 \times 5$$

$$\sigma = 11.5 \text{ (Ans.)}$$

PROBLEMS TO SOLVE

Find standard deviation:

1.

INT.	f
90–99	2
80–89	3
70–79	8
60–69	10
50–59	9
40–49	8
30–39	6
20–29	3
10–19	1

2.

INT.	f
90–99	3
80–89	7
70–79	8
60–69	9
50–59	12
40–49	10
30–39	7
20–29	5
10–19	2
0–9	1

3.

INT.	f
95–99	2
90–94	4
85–89	8
80–84	10
75–79	15
70–74	12
65–69	7
60–64	5
55–59	2
50–54	0
45–49	1

4.

INT.	f
75–79	2
70–74	4
65–69	4
60–64	8
55–59	12
50–54	9
45–49	7
40–44	4
35–39	0
30–34	2
25–29	0
20–24	1

5.

INT.	f
125–129	4
120–124	6
115–119	15
110–114	12
105–109	10
100–104	8
95–99	7
90–94	5
85–89	2
80–84	0
75–79	1

6.

INT.	f
82–83	4
80–81	0
78–79	7
76–77	8
74–75	15
72–73	10
70–71	5
68–69	5
66–67	4
64–65	2
62–63	2

7.

INT.	f
180–184	5
175–179	0
170–174	5
165–169	10
160–164	15
155–159	20
150–154	10
145–149	10
140–144	5
135–139	5
130–134	0
125–129	5

8.

INT.	f
90–92	3
87–89	5
84–86	4
81–83	0
78–80	5
75–77	8
72–74	7
69–71	5
66–68	10
63–65	12
60–62	10
57–59	7

9.

INT.	f
90–99	2
80–89	3
70–79	3
60–69	8
50–59	3
40–49	2
30–39	0
20–29	2
10–19	1
0–9	1

10

INT.	f
90–94	3
85–89	7
80–84	8
75–79	7
70–74	10
65–69	8
60–64	6
55–59	7
50–54	4

11.

INT.	f
115–119	1
110–114	2
105–109	0
100–104	7
95–99	10
90–94	5
85–89	8
80–84	7
75–79	2
70–74	7
65–69	1

12.

INT.	f
180–199	1
160–179	1
140–159	2
120–139	5
100–119	8
80–99	10
60–79	5
40–59	3
20–39	3
0–19	2

13.

INT.	f
120–129	2
110–119	4
100–109	6
90–99	8
80–89	10
70–79	12
60–69	5
50–59	2
40–49	1

14.

INT.	f
85–89	3
80–84	7
75–79	8
70–74	4
65–69	6
60–64	3
55–59	4
50–54	2

15.

INT.	f
135–149	2
120–134	3
105–119	4
90–104	3
75–89	12
60–74	6
45–59	3
30–44	2
15–29	2
0–14	3

16.

INT.	f
78–79	1
76–77	2
74–75	5
72–73	2
70–71	15
68–69	10
66–67	8
64–65	7
62–63	3
60–61	2

17.

INT.	f
18	1
17	3
16	2
15	4
14	10
13	8
12	5
11	3
10	2

18.

INT.	f
69–71	3
66–68	8
63–65	10
60–62	12
57–59	9
54–56	7
51–53	6
48–50	4
45–47	1

19.

INT.	f
180–199	3
160–179	7
140–159	8
120–139	10
100–119	12
80–99	9
60–79	8
40–59	6
20–39	3
0–19	2

20.

INT.	f
65–69	4
60–64	7
55–59	9
50–54	10
45–49	15
40–44	7
35–39	5
30–34	8
25–29	5

21.

INT.	f
42–43	2
40–41	4
38–39	0
36–37	4
34–35	5
32–33	9
30–31	8
28–29	12
26–27	10
24–25	8
22–23	7
20–21	6
18–19	3
16–17	2

22.

INT.	f
84–85	4
82–83	5
80–81	7
78–79	7
76–77	8
74–75	9
72–73	15
70–71	10
68–69	8
66–67	7
64–65	6
62–63	5
60–61	3

23.

INT.	f
98–99	3
96–97	4
94–95	8
92–93	12
90–91	15
88–89	10
86–87	9
84–85	6
82–83	2
80–81	0
78–79	0
76–77	4
74–75	0
72–73	1

24.

INT.	f
300–314	2
285–299	2
270–284	0
255–269	3
240–254	4
225–239	8
210–224	15
195–209	20
180–194	18
165–179	12
150–164	10
135–149	8
120–134	7
105–119	5
90–104	2

25.

INT.	*f*
195–209	4
180–194	4
165–179	5
150–164	8
135–149	15
120–134	20
105–119	22
90–104	17
75–89	12
60–74	10
45–59	8
30–44	4
15–29	0
0–14	1

26.

INT.	*f*
200–219	5
180–199	6
160–179	8
140–159	10
120–139	12
100–119	16
80–99	14
60–79	10
40–59	7
20–39	5
0–19	3

27.

INT.	*f*
95–99	4
90–94	0
85–89	8
80–84	14
75–79	16
70–74	20
65–69	20
60–64	15
55–59	9
50–54	6
45–49	5
40–44	3

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CHAPTER XVII

THE QUARTILE DEVIATION

Explanation. — The *quartile deviation* (Q) is one-half the interquartile range. The interquartile range is the difference between the upper and lower quartiles. The upper quartile (Q_3) is the point which marks off the highest one-fourth of the measures; the lower quartile (Q_1) is the point which marks off the lowest one-fourth of the measures.

The formula for the quartile deviation is

$$Q = \frac{Q_3 - Q_1}{2}$$

ILLUSTRATIVE PROBLEM. —

Find quartile deviation:

INT.	f
75–79	1
70–74	3
65–69	5
60–64	6
55–59	9
50–54	6
45–49	8
40–44	5
35–39	2
30–34	2
25–29	1

Solution:

INT.	f		
75–79	1		
70–74	3		$\frac{N}{4} = 12$
65–69	5	(9)	
60–64	6		$Q_3 = 65 - \frac{3}{6} \times 5 = 62.5$
55–59	9		
50–54	6		
45–49	8		$Q_1 = 45 + \frac{3}{8} \times 5 = 46.2$
40–44	5	(10)	$Q = \frac{Q_3 - Q_1}{2}$
35–39	2		
30–34	2		$Q = \frac{62.5 - 46.2}{2}$
25–29	1		$Q = 16.3 \div 2$
	N 48		$Q = 8.1$ (Ans.)

How to find the upper and lower quartiles. — The quartiles, Q_3 and Q_1, are found exactly like the median, except for the fact that we are separating one-fourth of the cases, instead of one-half of them.

To find the upper quartile (Q_3):

(*a*) Divide N by 4.

(*b*) Count one-fourth of the frequencies from the top to find the interval containing Q_3. (If N is 100, this will be the interval containing the 25th case; if N is 40, it will be the interval containing the 10th case, etc.)

(*c*) Find the sum of the frequencies above this Q_3 interval, and subtract from $\frac{N}{4}$. This gives the number yet to be used.

(*d*) Divide the number yet to be used by the frequency of the Q_3 interval, and multiply the quotient by the size of the interval.

(*e*) Subtract the result from the top of the Q_3 interval.

ILLUSTRATIVE PROBLEM, *in finding* Q_3. —

INT.	f	
90–99	1	
80–89	2	
70–79	4	(7)
60–69	8	
50–59	6	
40–49	6	
30–39	5	
20–29	3	
10–19	1	
	N 36	

$$\frac{N}{4} = 9$$

$$Q_3 = 70 - \tfrac{3}{8} \times 10$$

$$Q_3 = 70 - \tfrac{30}{8}$$

$$Q_3 = 70 - 2.5$$

$$Q_3 = 67.5$$

In locating the lower quartile (Q_1), the simplest method is to follow the plan of obtaining the median *by counting upward;* the only difference being that we are looking for $\frac{N}{4}$ instead of $\frac{N}{2}$.

To find the lower quartile:

(*a*) Divide N by 4.

(*b*) Count one-fourth of the frequencies from the bottom to find the interval containing Q_1.

(*c*) Find the sum of the frequencies below this Q_1 interval, and subtract from $\frac{N}{4}$. This gives the number yet to be used.

(*d*) Divide the number yet to be used by the frequency of the Q_1 interval, and multiply the quotient by the size of the interval.

(*e*) Add the result to the bottom of the Q_1 interval.

ILLUSTRATIVE PROBLEM, *in finding* Q_1. —

INT.	f	
90–99	1	
80–89	2	
70–79	4	
60–69	8	
50–59	6	
40–49	6	
30–39	5	
20–29	3	(4)
10–19	1	
	N 36	

$$\frac{N}{4} = 9$$

$$Q_1 = 30 + \tfrac{5}{5} \times 10$$

$$Q_1 = 40.0$$

ILLUSTRATIVE PROBLEM. —

Find Q:

INT.	f
190–199	3
180–189	3
170–179	6
160–169	8
150–159	9
140–149	10
130–139	10
120–129	5
110–119	2
100–109	1
	N 56

Solution:

INT.	f	
190–199	3	
180–189	3	
170–179	6	(12)
160–169	8	
150–159	7	
140–149	11	
130–139	10	
120–129	5	(8)
110–119	2	
100–109	1	
	N 56	

$$\frac{N}{4} = 14$$

$$Q_3 = 170 - \tfrac{2}{8} \times 10 = 167.5$$

$$Q_1 = 130 + \tfrac{6}{10} \times 10 = 136.0$$

Formula: $$Q = \frac{Q_3 - Q_1}{2}$$

$$Q = \frac{167.5 - 136.0}{2}$$

$$Q = 15.7 \text{ (Ans.)}$$

ILLUSTRATIVE PROBLEM. —

Find Q:

INT.	f
95–99	1
90–94	4
85–89	5
80–84	12
75–79	10
70–74	10
65–69	8
60–64	7
55–59	2
50–54	0
45–49	0
40–44	2
35–39	1
N	62

Solution:

INT.	f	
95–99	1	
90–94	4	
85–89	5	(10)
80–84	12	
75–79	10	
70–74	10	
65–69	8	
60–64	7	(12)
55–59	2	
50–54	0	
45–49	0	
40–44	2	
35–39	1	
N	62	

$$\frac{N}{4} = 15.5$$

$$Q_3 = 85 - \frac{5.5}{12} \times 5 = 82.7$$

$$Q_1 = 65 + \frac{3.5}{8} \times 5 = 67.1$$

Formula: $$Q = \frac{Q_3 - Q_1}{2}$$

$$Q = \frac{82.7 - 67.1}{2}$$

$$Q = 15.6 \div 2$$

$$Q = 7.8 \text{ (Ans.)}$$

PROBLEMS TO SOLVE

Find quartile deviation:

1.

INT.	f
90–99	1
80–89	2
70–79	6
60–69	5
50–59	6
40–49	8
30–39	4
20–29	4
10–19	3
0–9	1
N	40

2.

INT.	f
70–74	1
65–69	2
60–64	5
55–59	7
50–54	7
45–49	10
40–44	10
35–39	6
30–34	6
25–29	3
20–24	2
15–19	1

3.

INT.	f
95–99	2
90–94	4
85–89	12
80–84	10
75–79	12
70–74	16
65–69	8
60–64	8
55–59	6
50–54	2

4.

INT.	f
110–119	2
100–109	3
90–99	5
80–89	7
70–79	13
60–69	15
50–59	15
40–49	0
30–39	10
20–29	5
10–19	2
0–9	3

5.

INT.	f
100–109	3
90–99	5
80–89	7
70–79	7
60–69	8
50–59	7
40–49	5
30–39	4
20–29	3
10–19	2
0–9	1

6.

INT.	f
100–104	2
95–99	2
90–94	2
85–89	3
80–84	4
75–79	2
70–74	5
65–69	6
60–64	6
55–59	5
50–54	7
45–49	3
40–44	1

7.

INT.	f
120–124	1
115–119	0
110–114	3
105–109	2
100–104	5
95–99	7
90–94	10
85–89	6
80–84	6
75–79	5
70–74	3

8.

INT.	f
220–239	5
200–219	5
180–199	8
160–179	8
140–159	0
120–139	11
100–119	12
80–99	11
60–79	12
40–59	8
20–39	8

9.

INT.	f
200–209	1
190–199	1
180–189	2
170–179	8
160–169	6
150–159	10
140–149	9
130–139	7
120–129	6
110–119	2

10.

INT.	f
80–81	5
78–79	5
76–77	7
74–75	10
72–73	15
70–71	13
68–69	30
66–67	25
64–65	5
62–63	3
60–61	2

11.

INT.	f
320–339	1
300–319	2
280–299	2
260–279	4
240–259	8
220–239	11
200–219	9
180–199	7
160–179	10
140–159	5
120–139	1

12.

INT.	f
95–99	5
90–94	5
85–89	5
80–84	7
75–79	10
70–74	14
65–69	16
60–64	21
55–59	8
50–54	9
45–49	6
40–44	4

13.

INT.	f
95–99	1
90–94	2
85–89	3
80–84	6
75–79	9
70–74	15
65–69	11
60–64	3
55–59	8
50–54	9
45–49	5

14.

INT.	f
200–219	1
180–199	2
160–179	6
140–159	8
120–139	11
100–119	12
80–99	7
60–79	6
40–59	4
20–39	2
0–19	1

15.

INT.	f
89	2
88	3
87	4
86	3
85	6
84	8
83	6
82	3
81	4
80	1

16.

INT.	f
95–99	1
90–94	2
85–89	0
80–84	2
75–79	5
70–74	9
65–69	11
60–64	5
55–59	4
50–54	0
45–49	1

17.

INT.	f
120–129	1
110–119	0
100–109	2
90–99	5
80–89	9
70–79	12
60–69	6
50–59	3
40–49	3
30–39	2
20–29	1

18.

INT.	f
85–89	1
80–84	2
75–79	2
70–74	4
65–69	3
60–64	6
55–59	8
50–54	4
45–49	10
40–44	4
35–39	3
30–34	1

19.

INT.	f
90–99	2
80–89	4
70–79	8
60–69	10
50–59	15
40–49	9
30–39	0
20–29	3
10–19	3
0–9	2

20.

INT.	f
95–99	2
90–94	4
85–89	4
80–84	9
75–79	6
70–74	8
65–69	7
60–64	4
55–59	3
50–54	2
45–49	1

21.

INT.	f
90–99	3
80–89	4
70–79	6
60–69	4
50–59	0
40–49	9
30–39	8
20–29	5
10–19	5
0–9	2

22.

INT.	f
220–239	1
200–219	3
180–199	5
160–179	6
140–159	10
120–139	14
100–119	18
80–99	4
60–79	4
40–59	3

23.

INT.	f
80–89	3
70–79	5
60–69	15
50–59	17
40–49	12
30–39	10
20–29	4
10–19	3
0–9	1

24.

INT.	f
200–219	3
180–199	4
160–179	5
140–159	7
120–139	8
100–119	10
80–99	12
60–79	7
40–59	5
20–39	4

25.

INT.	*f*
95–99	8
90–94	6
85–89	5
80–84	14
75–79	23
70–74	20
65–69	8
60–64	8
55–59	0
50–54	2
45–49	1

26.

INT.	*f*
95–99	2
90–94	3
85–89	6
80–84	8
75–79	10
70–74	11
65–69	14
60–64	9
55–59	5
50–54	3
45–49	2
40–44	1

27.

INT.	*f*
50–54	2
45–49	3
40–44	5
35–39	15
30–34	14
25–29	12
20–24	10
15–19	7
10–14	4
5–9	3
0–4	1

28.

INT.	*f*
220–239	2
200–219	2
180–199	3
160–179	4
140–159	6
120–139	12
100–119	22
80–99	12
60–79	6
40–59	4
20–39	3
0–19	2

29.

INT.	*f*
65–69	2
60–64	3
55–59	5
50–54	8
45–49	10
40–44	12
35–39	10
30–34	5
25–29	7
20–24	4
15–19	3
10–14	1

30.

INT.	*f*
90–91	1
88–89	2
86–87	3
84–85	3
82–83	5
80–81	16
78–79	20
76–77	17
74–75	8
72–73	5
70–71	3
68–69	2

31.

INT.	f
280–299	2
260–279	6
240–259	3
220–239	10
200–219	10
180–199	7
160–179	20
140–159	2
120–139	8
100–119	5
80–99	4
60–79	3

32.

INT.	f
115–119	5
110–114	7
105–109	9
100–104	3
95–99	8
90–94	23
85–89	17
80–84	7
75–79	7
70–74	4

33.

INT.	f
380–399	3
360–379	6
340–359	8
320–339	10
300–319	12
280–299	12
260–279	11
240–259	9
220–239	7
200–219	6
180–199	3
160–179	1

34.

INT.	f
90–94	3
85–89	8
80–84	10
75–79	20
70–74	15
65–69	10
60–64	13
55–59	3
50–54	0
45–49	0
40–44	2

35.

INT.	f
220–239	2
200–219	3
180–199	5
160–179	12
140–159	8
120–139	16
100–119	10
80–99	17
60–79	0
40–59	5
20–39	4

36.

INT.	f
100–109	6
90–99	12
80–89	7
70–79	8
60–69	15
50–59	3
40–49	6
30–39	9
20–29	8
10–19	3

37.

INT.	f
110–119	4
100–109	5
90–99	6
80–89	8
70–79	10
60–69	20
50–59	15
40–49	7
30–39	6
20–29	5
10–19	5
0–9	5

38.

INT.	f
90–94	5
85–89	5
80–84	6
75–79	6
70–74	5
65–69	8
60–64	15
55–59	10
50–54	6
45–49	0
40–44	1
35–39	0
30–34	2

39.

INT.	f
220–239	2
200–219	3
180–199	4
160–179	5
140–159	6
120–139	9
100–119	20
80–99	14
60–79	10
40–59	9
20–39	6
0–19	4

40.

INT.	f
100–109	2
90–99	5
80–89	8
70–79	14
60–69	20
50–59	23
40–49	15
30–39	10
20–29	9
10–19	8
0–9	6

41.

INT.	f
95–99	2
90–94	3
85–89	5
80–84	4
75–79	6
70–74	9
65–69	7
60–64	8
55–59	3
50–54	1
45–49	1
40–44	0
35–39	4
30–34	6
25–29	2

42.

INT.	f
90–99	1
80–89	2
70–79	3
60–69	5
50–59	8
40–49	9
30–39	6
20–29	4
10–19	2
0–9	1

43.

INT.	f
30–31	2
28–29	4
26–27	8
24–25	11
22–23	14
20–21	18
18–19	9
16–17	12
14–15	8
12–13	7
10–11	4
8–9	3

44.

INT.	f
95–99	3
90–94	5
85–89	7
80–84	9
75–79	7
70–74	12
65–69	18
60–64	20
55–59	11
50–54	6
45–49	3
40–44	3

45.

INT.	f
120–129	2
110–119	4
100–109	5
90–99	6
80–89	8
70–79	10
60–69	12
50–59	19
40–49	7
30–39	6
20–29	3
10–19	3

46.

INT.	f
95–99	2
90–94	0
85–89	4
80–84	5
75–79	8
70–74	13
65–69	15
60–64	14
55–59	20
50–54	24
45–49	20
40–44	18
35–39	12
30–34	8
25–29	10
20–24	7
15–19	0
10–14	4
5–9	3
0–4	1

47.

INT.	f
74–75	3
72–73	6
70–71	6
68–69	5
66–67	0
64–65	4
62–63	3
60–61	8
58–59	12
56–57	7
54–55	9
52–53	10
50–51	8
48–49	13
46–47	14
44–45	13
42–43	12
40–41	10
38–39	10
36–37	8
34–35	9
32–33	7
30–31	4
28–29	2
26–27	1

48.

INT.	f
48	3
47	0
46	0
45	5
44	7
43	8
42	10
41	10
40	12
39	14
38	10
37	7
36	8
35	10
34	12
33	15
32	8
31	8
30	4
29	0
28	4
27	3
26	5
25	2
24	1

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McCall, *How to Measure in Education*, p. 380.
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PART IV

MEASURES OF RELATIONSHIP OR CORRELATION

CHAPTER XVIII

THE MEANING OF CORRELATION

Explanatory definition. — By correlation is meant the relationship, or statistical resemblance, of parallel measurements of the same individuals. It is a measure of the degree of resemblance or difference. It tells to what extent the measures tend to vary together. Most correlation problems deal with two measured series between which there is some supposed relationship, as in the case of height and weight. If tall people are mostly heavy, and short people mostly light, we may say that there is a *positive* relationship between height and weight. But if tall people were usually lighter than short people, and short people heavier than tall people, we would say that the relationship between height and weight is inverse, or *negative*. If tallness and heaviness seemed to have no connection at all, and weights and heights were entirely independent of each other, we would have an example of the absence of correlation. In terms of correlation procedure the three situations just referred to would be expressed respectively as + (positive correlation), − (negative correlation), and 0 (no correlation, or zero correlation).

But we usually wish to know, not only whether the relationship is positive or negative, but *just to what extent* the relationship exists. This we find in the coefficient of correlation, which is the answer to a correlation problem. Coefficients range from −1.00 to +1.00. This is a wide range, of about 200 points, for we usually carry out the answer to two places

beyond the decimal point. The coefficient +1.00 represents a perfect positive correlation, indicating unvarying relationship between the things measured. The coefficient −1.00 represents a perfect negative correlation, indicating that the things measured bear an absolutely inverse relationship to each other. Other coefficients, such as +.93, +.75, +.50, +.03, −.15, −.45, etc., indicate the extent to which the correlation approaches either of these extremes. A correlation of +.95 is obviously much nearer perfect than one of +.83; while one of −.02 is so near zero as to indicate practically no relationship at all.

Significance of correlation coefficients. — In determining the relationship between two series of measures, in terms of correlation, we have, in the coefficient, an index which should tell us what we want to know. Inasmuch as +1.00 means a perfect relationship, and perfect relationships almost never occur in actual practice, we may attach a considerable degree of importance to coefficients as high as +.90 or even +.80. In fact we usually think of a correlation as being high if it exceeds +.70. Certainly one as low as +.10 would be low, and we might think of +.50 as being medium, since it is half-way between 0 and +1.00.

Numerous attempts have been made to generalize on the significance of correlation coefficients, but it appears unsafe to do so, because of the different settings in which correlation measures are used. There are some situations in which a correlation of +.56 might be high; that is, it would certainly be high in comparison with +.24, or +.35, or +.43. On the other hand, in most cases we would not consider +.56 as having great significance.

The experience of Rugg has led him to regard coefficients of correlation as having general significance as follows:

Above +.60 or +.70, " high."
+.35 or +.40 to +.50 or +.60, " markedly present."
+.15 or +.20 to +.35 or +.40, " present but low."
less than +.15 or +.20, " low."

Regarding these and similar interpretations, Odell suggests that " the use of adjectives is decidedly unsatisfactory and often meaningless unless they are employed in a definitely limited situation." He advises against the use of such generalizations. Correlation coefficients are " high " or " low " only on a relative basis.

Practical uses of correlation. — When we measure the relationship between two things, we have a basis for some conclusion as to the existence of common causal factors. Odell cites the following sizes of coefficients of correlation usually found in studies of education:

Cost of instruction and total cost of education	+.90 to +.95
First and second applications of an individual intelligence test	+.90 to +.95
First and second applications of a group test	+.60 to +.90
School marks in language and science	+.40 to +.70
School marks in language and manual arts	+.30 to +.50
Handwriting and intelligence	.00 to +.10

These are all positive correlations (excepting the .00) and hence indicate that some relationship, although small in some instances, exists between each set of things measured. In the case of the first and second applications of an intelligence test, in which the

correlations range from +.90 to +.95, we have in these figures sufficient evidence to warrant the conclusion that a person's score on a second testing will not differ much from the score of the first test. In the case of handwriting and intelligence, even these coefficients are positive, indicating that there is probably some relationship between them. However, the relationship is so small as to be of little practical significance, and we certainly would not be justified, in the face of such figures, to infer anything about a person's intelligence from the quality of his handwriting, or *vice versa*.

Types of correlation measures. — Two methods of correlation will be illustrated in the following chapters: (1) the rank-difference method, which we shall use when dealing with simple series of scores, and (2) the product-moment method, which we shall apply when the measures are arranged in a frequency distribution. The rank-difference method, as its name implies, is based on differences in rank, or relative position, of the scores. It is usually limited to problems having a small number of cases. The product-moment method is a more complex procedure, based on deviations from the central tendency. It is applicable, however, to any number of cases, and is therefore in more extensive use. Both methods should be known to all persons having occasion to use or interpret statistical facts.

Special methods of correlation. — The increasing importance of correlation procedure in many forms of work has caused a number of statisticians to develop special devices in the form of correlation charts. In the texts of Kelley and Otis these charts are explained, and will be of use to many students and statistical

workers. The methods illustrated here are not to be construed as being necessarily the best or quickest, but are adapted to beginners, and should serve as points of departure in case one of the special methods is learned at a subsequent time.

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CHAPTER XIX

CORRELATION BY THE RANK-DIFFERENCE METHOD

Explanation. — The rank-difference, or Spearman, method of correlation is used with simple series of scores, usually when the number of cases is less than about 30. It is based on the differences in rank, or relative position, of the scores. The Greek letter *rho* (ρ) is used to designate the coefficient. The formula is

$$\rho = 1 - \frac{6\,\Sigma D^2}{N(N^2 - 1)}$$

in which ΣD^2 is the sum of the squared differences in rank, and N is the number of cases.

Inasmuch as +1.00 represents a perfect correlation, and we subtract from 1.00 to find ρ, the answer to any problem will depend upon the amount of difference, or unlikeness, between the ranks of the two things measured; the greater the difference, the lower the correlation. The lowest possible correlation is −1.00, for $6\,\Sigma D^2$ can never be greater than twice $N\,(N^2 - 1)$.

The rank-difference method is relatively simple, and will be found valuable for determining the approximate relationship between two tests or measures applied to a small group. Practice in this method also affords an opportunity to understand the meaning and significance of correlation.

How to rank. — The ranking of scores means numbering them according to the positions they would take if they were arranged in statistical order in a ver-

tical column. The highest score is ranked 1, the next highest 2, the next 3, and so on, the lowest rank being given to the lowest score. It is not necessary to rearrange the measures to rank them, for we can pick them out, one at a time, in order of decreasing value. Here are two sets of scores, called "x" and "y" repectively, which have been given their proper ranks:

x		y	
SCORE	RANK	SCORE	RANK
87	4	52	7
92	3	75	3
64	6	73	4
45	10	70	5
93	2	88	2
98	1	45	9
75	5	69	6
52	7	90	1
50	8	43	10
48	9	47	8

It frequently happens that the same score appears twice or more often in a series. In such cases it is necessary to divide the ranks equally among them. For example:

SCORE	RANK
92	5
98	2
95	3.5
99	1
87	6
95	3.5

In this case the score 99 is highest, so it is given a rank of 1; 98 is ranked 2; but the next highest score,

95, occurs twice. So we take the next two ranks, 3 and 4, and give each 95 the average, 3.5. Inasmuch as we have now used ranks 3 and 4, the next highest score, 92, is ranked 5, and the next score, 87, is ranked 6.

When a score appears three or more times, we follow the same procedure, giving it the average of the ranks which its numbers occupy. For example:

SCORE	RANK		SCORE	RANK	
54	7		65	1	
58	5		62	2	
65	1	which may be	60	3	
50	8.5	explained by	58	(4)	
60	3	arranging the	58	(5)	Average, 5
58	5	scores in order,	58	(6)	
47	10	thus:	54	7	
50	8.5		50	(8)	Average, 8.5
62	2		50	(9)	
58	5		47	10	

ILLUSTRATIVE PROBLEM. —

Rank the following scores:

58
62
97
74
98
96
59
85

Solution:

SCORE	RANK
58	8
62	6
97	2
74	5
98	1
96	3
59	7
85	4

ILLUSTRATIVE PROBLEM. — *Solution:*

Rank the following scores:

	SCORE	RANK
78	78	6
75	75	8
88	88	1
72	72	10
80	80	4.5
70	70	11
80	80	4.5
75	75	8
83	83	3
75	75	8
68	68	12
85	85	2

ILLUSTRATIVE PROBLEM. — *Solution:*

Rank the following scores:

	(Same order)	
38	SCORE	RANK
32	38	5
45	32	8.5
30	45	1
32	30	11.5
35	32	8.5
43	35	6
40	43	2
32	40	3.5
30	32	8.5
40	30	11.5
32	40	3.5
20	32	8.5
25	20	14
	25	13

PROBLEMS TO SOLVE

Rank each column:

1.	2.	3.	4.	5.	6.
90	85	35	19	215	30
75	94	43	18	235	25
98	82	45	16	205	35
93	93	32	20	200	36
80	95	40	15	210	20
70	90	43	18	200	30
97	100	30	13	220	35
87	90	25	18	215	18
95	83	38	13	200	30
85	80	32	14	195	18
	83	28	18	225	36
	87	32	15	220	30
		28		230	30

Outline of procedure. — In solving a correlation problem, we are dealing with two sets of scores; that is, parallel series of measures. These are usually termed "x" and "y," respectively. If we wished to determine the correlation between height and weight, x could represent height and y weight, or *vice versa*, providing we are consistent after deciding. Unless the correlation is perfect, which is rarely the case, the ranks for the x series will be different from those of the y series. The procedure is as follows:

(*a*) Write the scores in two parallel vertical columns, keeping each pair of scores together. Call one series x, the other y.

(*b*) Rank the x scores.

(*c*) Rank the y scores.

(*d*) Find the difference between each pair of ranks. Write the differences in a column headed D (difference).

(*e*) Square each difference, and list these in a column headed D^2.

(*f*) Add the D^2 column. The sum is ΣD^2.

(*g*) Multiply ΣD^2 by 6. (This is a constant procedure for all problems.)

(*h*) Divide 6 ΣD^2 by $N(N^2 - 1)$.

(*i*) Subtract from 1.00. The result will be the coefficient of correlation, *rho*.

Formula:

$$\rho = 1 - \frac{6\,\Sigma D^2}{N\,(N^2 - 1)}$$

ILLUSTRATIVE PROBLEM. — Find *rho* for the following measures:

NO.	x	y
1.	75	42
2.	65	24
3.	62	22
4.	73	35
5.	78	40
6.	68	27
7.	76	25
8.	64	32
9.	70	30
10.	72	38

Solution:

MEASURES		RANKS			
x	y	x	y	D	D^2
75	42	3	1	2	4
65	24	8	9	1	1
62	22	10	10	0	0
73	35	4	4	0	0
78	40	1	2	1	1
68	27	7	7	0	0
76	25	2	8	6	36
64	32	9	5	4	16
70	30	6	6	0	0
72	38	5	3	2	4
N	10				62 (ΣD^2)

Formula:

$$\rho = 1 - \frac{6\,\Sigma D^2}{N\,(N^2 - 1)}$$

$$\rho = 1 - \frac{6 \times 62}{10\,(10^2 - 1)}$$

$$\rho = 1 - \tfrac{372}{990}$$

$$\rho = 1 - .37$$

$$\rho = +.63 \text{ (Ans.)}$$

ILLUSTRATIVE PROBLEM. — Find the correlation between height and weight by the rank-difference method, from the following measurements of ten boys:

NO.	HT.	WT.
1.	67	90
2.	70	85
3.	58	140
4.	65	88
5.	62	150
6.	72	78
7.	66	80
8.	68	125
9.	60	75
10.	63	80

Solution:

MEASURES		RANKS			
x	y	x	y	D	D^2
67	90	4	4	0	0
70	85	2	6	4	16
58	140	10	2	8	64
65	88	6	5	1	1
62	150	8	1	7	49
72	78	1	9	8	64
66	80	5	7.5	2.5	6.25
68	125	3	3	0	0
60	75	9	10	1	1
63	80	7	7.5	.5	.25
N	10				201.50 (ΣD^2)

Formula:

$$\rho = 1 - \frac{6\,\Sigma D^2}{N\,(N^2 - 1)}$$

$$\rho = 1 - \frac{6 \times 201.5}{10 \times (10^2 - 1)}$$

$$\rho = 1 - \frac{1209}{990}$$

$$\rho = 1 - 1.22$$

$$\rho = -\ .22 \text{ (Ans.)}$$

ILLUSTRATIVE PROBLEM. — Find *rho:*

NO.	x	y
1.	43	65
2.	58	79
3.	45	65
4.	50	80
5.	56	90
6.	40	75
7.	44	63
8.	50	75
9.	48	88
10.	55	85
11.	35	70
12.	39	65

Solution:

MEASURES		RANKS			
x	y	x	y	D	D^2
43	65	9	10	1	1
58	79	1	5	4	16
45	65	7	10	3	9
50	80	4.5	4	.5	.25
56	90	2	1	1	1
40	75	10	6.5	3.5	12.25
44	63	8	12	4	16
50	75	4.5	6.5	2	4
48	88	6	2	4	16
55	85	3	3	0	0
35	70	12	8	4	16
39	65	11	10	1	1
N	12				92.50 (ΣD^2)

Formula:

$$\rho = 1 - \frac{6\,\Sigma D^2}{N\,(N^2 - 1)}$$

$$\rho = 1 - \frac{6 \times 92.5}{12\,(12^2 - 1)}$$

$$\rho = 1 - \tfrac{555}{1716}$$

$$\rho = 1 - .32$$

$$\rho = +.68 \text{ (Ans.)}$$

The probable error. — After a coefficient of correlation is computed, it is good statistical practice to compute a supplementary measure, called the *probable error* (*P.E.*). This is a measure of variability, corresponding to the standard deviation, quartile deviation, etc., and, when used in correlation, tells the extent to

which the result may be depended upon as being a true index. It is not an " error " in the usual meaning of the word, and does not imply that the answer is a mistake. It merely helps to indicate the significance of the correlation. It is usually considered that a coefficient of correlation should be at least four times its probable error to be significant.

For the rank-difference method of correlation the formula for the probable error is

$$P.E.\ \rho = .706\frac{1 - \rho^2}{\sqrt{N}}$$

Thus, in a problem dealing with 100 cases, in which ρ is $+.50$, the calculation of the probable error would be as follows:

$$P.E.\ \rho = .706\frac{1 - .50^2}{\sqrt{100}}$$

$$P.E.\ \rho = .706\frac{1 - .25}{10}$$

$$P.E.\ \rho = .706 \times .075$$

$$P.E.\ \rho = .052 \text{ (Ans.)}$$

The probable error is written with the accompanying coefficient of correlation, thus:

$$\rho = +.50 \pm .052$$

It is not necessary to find the probable error in calculating a coefficient of correlation, but in practical work it will be found a valuable supplementary measure.

PROBLEMS TO SOLVE

Find the correlation by the rank-difference method:

1.

x	y
90	85
80	88
88	98
83	95
81	91
89	99
75	80
78	90
85	92
73	83

2.

x	y
100	68
96	75
89	70
84	66
98	72
80	53
95	74
87	60
82	54
90	55

3.

x	y
68	45
58	35
63	42
52	31
67	46
55	30
64	40
60	38
56	41
65	32

4.

x	y
75	87
74	95
73	99
70	92
68	82
67	80
65	96
62	75
60	90
59	84

5.

x	y
85	12
75	16
72	24
82	13
79	10
70	25
80	18
67	15
74	11
65	20

6.

x	y
45	88
37	90
30	80
38	94
44	98
32	90
40	96
35	85
40	93
28	83

7.

x	y
68	48
67	45
61	38
73	43
62	47
60	35
70	50
58	30
61	40
65	47
55	32
58	33

8.

x	y
78	18
53	5
68	10
86	28
58	7
82	22
70	12
85	25
75	15
80	20
60	8
83	24
65	9
73	13
82	22

9.

x	y
93	42
91	50
90	44
88	49
85	48
84	80
82	35
80	38
75	28
72	33
70	30
68	27
65	20
60	25

10.

x	y
92	79
100	85
78	50
98	84
90	76
97	75
76	73
88	65
75	70
96	82
85	60
94	80

11.

x	y
80	75
92	78
100	75
55	50
84	73
75	60
95	81
88	74
71	72
82	64

12.

x	y
98	90
100	84
94	75
99	74
85	76
90	91
84	84
74	65
85	74
89	80
70	45
72	54

13.

x	y
100	84
96	74
95	85
75	60
84	75
50	44
74	69
94	83
98	98
92	90

14.

x	y
64	55
75	64
89	79
90	80
100	85
75	83
98	90
94	84
70	65
99	88

15.

x	y
95	84
100	90
75	64
99	89
84	80
88	75
94	70
90	85
74	70
65	69
78	84
67	59

16.

x	y
99	84
96	79
84	74
79	70
83	89
100	82
91	85
75	69
73	72
64	60
59	54
44	49

17.

x	y
99	84
100	100
96	82
85	80
99	79
94	93
95	95
74	70
78	78
65	72

18.

x	y
48	5
52	8
63	17
49	10
53	14
50	15
62	20
58	16
59	19
61	18
60	13
56	12

19.

x	y
18	46
21	54
27	92
20	91
24	87
16	90
26	58
30	65
15	72
28	93

20.

x	y
91	48
88	40
80	36
96	50
85	38
75	30
99	54
90	45
87	39
93	49

21.

x	y
75	74
63	68
70	100
87	60
89	95
94	70
90	83
76	99
88	56
95	80

22.

x	y
75	79
80	84
94	74
55	40
100	75
95	64
82	90
85	78
99	82
70	69
64	60
79	71
78	62
64	59
55	50

23.

x	y
85	54
73	52
99	63
84	70
96	51
48	78
63	98
54	90
70	83
62	85

24.

x	y
84	70
95	80
100	79
94	85
99	91
89	89
79	65
82	80
69	60
59	54
64	63
90	84
50	45
75	74
85	77

25.

x	y
95	84
75	79
84	80
89	85
99	89
76	70
100	95
97	90
74	75
80	60
92	83
91	91
72	71
65	69
60	50

26.

x	y
99	90
100	84
93	89
89	80
85	85
79	74
70	69
84	81
92	91
74	70
69	60
60	54
68	61
76	71
61	59

27.

x	y
77	21
93	15
75	27
65	35
87	19
70	30
99	13
76	25
90	17
80	20

28.

x	y
87	118
93	128
120	125
95	132
87	116
123	126
93	134
83	115
118	130
92	123
80	108
90	120

29.

x	y
87	12
80	15
92	18
93	11
75	10
74	9
76	7
88	14
67	8
77	16
70	13
85	6

30.

x	y
14	19
15	18
10	18
8	14
23	26
29	31
20	39
22	41
18	57
18	38
20	30
24	12
21	36
13	32
7	12

31.

x	y
70	58
75	50
80	63
60	42
97	68
72	47
62	35
99	65
65	57
83	52
50	30
88	60
52	40
75	55
74	48
85	64
55	45
75	59
58	53
85	51

32.

x	y
118	79
111	77
71	92
104	88
81	99
85	100
111	86
107	92
77	117
67	106
76	118
101	92
102	86
74	106
85	89
90	102
67	130
90	100
125	92

33.

x	y
80	35
67	22
63	25
57	20
48	27
45	25
78	46
64	38
82	48
75	30
71	35
65	20
72	35
32	29
50	24
60	26
75	40
35	28
70	40
55	21
85	45
40	18
76	44
70	30
53	20
74	35
84	47
68	27
77	42
75	29

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CHAPTER XX

CORRELATION BY THE PRODUCT-MOMENT METHOD

Explanation. — The essential difference between the rank-difference and product-moment methods is that the latter may be used with frequency distributions, and hence is not limited to problems having a small number of cases. The method of procedure involves, first of all, placing the measures in a two-way distribution. As in the case of the rank-difference method, we have two measured series, called x and y.

We now have a practical use for the standard deviation, which is calculated as shown in Chapter XVI, except for the fact that in a two-way distribution we do not multiply by the size of the interval.

The product-moment method involves the following general steps: (*a*) arranging the measures in the two-way distribution, using intersecting intervals; (*b*) calculating the standard deviation of each of the two sets of frequencies; (*c*) calculating the XY values, by using the product of the two deviations which this arrangement gives to each frequency.

This method was developed by Professor Karl Pearson, of the University of London, and is the most widely used correlation procedure. Numerous variations of the method have been devised, but we shall study a form which appears to be most easily understood and followed by beginners.

The two-way distribution. — This type of arrangement is merely two superimposed frequency distributions, thus:

x series

y series	60–64	65–69	70–74	75–79	80–84	85–89	90–94	95–99	$f(y)$
90–99				2	3	1			6
80–89		1	2	3	2	2			10
70–79		3	4	5	2	4	3		21
60–69	2		1	6	3	2	2		16
50–59	1	2	1	4	2	3		1	14
40–49	1		1	2		2			6
$f(x)$	4	6	9	22	12	14	5	1	73

The two-way distribution is occasioned by the fact that each case has been measured for two things, x and y. Instead of making two separate distributions, we place them in one, containing the intervals for both series. N is 73, which is the total of each set of frequencies, x and y.

To make a two-way frequency distribution we must first study our data, and determine for each series what size of interval to use. We need not use the same size for both series, and the number of intervals need not be the same. Note in the preceding illustration that the x intervals are in fives, and the y intervals in tens. Also, there are 8 intervals in the x series, and 6 in the y series.

In correlation work we often condense the distributions to a smaller number of intervals than we might

use for other statistical measures. In doing so the result is not likely to be much affected, and the amount of labor is decreased. However, the procedure is the same, no matter what size the distributions may be.

The procedure for making a two-way distribution follows:

(*a*) Study the diagram on the preceding page. Note its arrangement.

(*b*) Select a suitable size of interval for each series. Keep in mind condensing the data, and at the same time your own convenience in tabulating.

(*c*) Draw a rough diagram, after the plan illustrated on the previous page. The intervals for the x series will be at the top, the lowest interval at the left. The y intervals will be arranged in a vertical column, just as most frequency distributions are arranged.

(*d*) Read the two scores for each case, and indicate the values by a tally mark in the space where the two respective intervals intersect.

(*e*) Total the frequencies for each interval in both series, thus obtaining N, which should correspond to the number of cases in the original data, and should be the total of both series of frequencies. (If it should not, the table should be rechecked, and the distributions made over, if necessary.)

ILLUSTRATIVE PROBLEM. — Arrange the following scores in a two-way distribution:

x	y	x	y	x	y	x	y
45	82	15	32	48	93	33	72
47	83	25	63	27	52	43	95
36	68	32	45	18	46	27	72
39	85	23	80	35	55	34	92
36	64	55	92	27	54	25	65
32	58	45	52	45	75	33	64
18	47	27	43	50	80	37	82
46	72	36	62	36	42	35	75
43	65	37	54	45	67	35	65
35	64	56	92	35	82	27	50

Solution. — Let us put both series in intervals of 10. The x intervals will be 10–19 to 50–59; the y intervals will be 30–39 to 90–99.

y \ x	10–19	20–29	30–39	40–49	50–59	$f(y)$
90–99			/	//	//	5
80–89		/	///	//	/	7
70–79		/	//	//		5
60–69		//	~~////~~	//		10
50–59		///	///	/		7
40–49	//	/	//			5
30–39	/					1
$f(x)$	3	8	17	9	3	40

ILLUSTRATIVE PROBLEM. — Arrange the following scores in a two-way distribution:

x	y	x	y	x	y	x	y	x	y
56	83	54	65	62	82	64	90	45	73
63	75	55	75	55	73	45	65	55	85
58	62	58	58	65	74	56	85	48	52
53	71	56	78	50	70	57	92	45	55
65	92	42	47	55	80	65	73	62	85
48	55	65	93	52	57	57	67	55	76
62	83	52	65	56	84	52	81	52	65
57	77	56	84	60	80	58	75	67	84
43	42	55	75	62	75	52	75	63	78
62	75	59	63	60	60	62	78	45	65

Solution. — Here the range of the x scores suggests intervals of 5; the range of the y scores suggests intervals of 10 (to keep the tables to a size approximating those in the preceding illustrations).

y \ x	40–44	45–49	50–54	55–59	60–64	65–69	f_y
90–99				/	/	//	4
80–89			/	~~////~~ /	////	/	12
70–79		/	///	~~////~~ //	~~////~~	//	18
60–69		//	///	///	/		9
50–59		///	/	/			5
40–49	//						2
f_x	2	6	8	18	11	5	50

Locating the two mid-intervals. — The next step in computing the correlation is marking out the two mid-intervals. For the preceding illustrative problem this would make the table look as follows:

		x						
		40–44	45–49	50–54	55–59	60–64	65–69	f_y
y	90–99				1	1	2	4
	80–89			1	6	4	1	12
	70–79		1	3	7	5	2	18
	60–69		2	3	3	1		9
	50–59		3	1	1			5
	40–49	2						2
	f_x	2	6	8	18	11	5	50

NOTE. — The tally marks have been changed to numbers. This may be done by marking the numbers over the tally marks with a heavy or colored pencil.

The x mid-interval is 55–59.

The y mid-interval is 70–79.

The remaining steps. — After locating the mid-intervals, it is necessary to extend the table at the right and the bottom to make the columns used in computing the standard deviation (as explained in Chapter 16). For the y frequencies, this will look the same as any regular problem; but for the x series the frequency distribution is in a horizontal position, with the higher intervals to the right of the mid-interval, and the lower

intervals to the left. The procedure for computing fd and fd^2, however, is otherwise the same.

Here is the table, showing how it looks when we obtain the necessary data to compute the standard deviation (σ) for each series:

		x									
		40–44	45–49	50–54	55–59	60–64	65–69	f_y	d_y	fd_y	fd^2_y
y	90–99				1	1	2	4	2	+8	16
	80–89			1	6	4	1	12	1	+12	12
	70–79		1	3	7	5	2	18	0		
	60–69		2	3	3	1		9	−1	−9	9
	50–59		3	1	1			5	−2	−10	20
	40–49	2						2	−3	−6	18
	f_x	2	6	8	18	11	5	50 N		−5 Σfd_y	75 Σfd^2_y
	d_x	−3	−2	−1	0	1	2				
	fd_x	−6	−12	−8		+11	+10	−5 Σfd_x			
	fd^2_x	18	24	8		11	20	81 Σfd^2_x			

We now have the following data:

$$N = 50$$
$$\Sigma fd_x = -5$$
$$\Sigma fd^2_x = 81$$
$$\Sigma fd_y = -5$$
$$\Sigma fd^2_y = 75$$

Finding the XY values. — It is now necessary to compute a set of values which we call XY. It is done by multiplying each frequency with the product of its two deviations: $f \times d \times d$. Here is the same table with the XY values indicated in each interval, and the new column at the right for computing XY, the sum of these values:

				x							
	40–44	45–49	50–54	55–59	60–64	65–69	f_y	d_y	fd_y	fd^2_y	XY
90–99				1	(+2) 1	(+8) 2	4	2	+8	16	+10
80–89			(−1) 1	6	(+4) 4	(+2) 1	12	1	+12	12	+5
y 70–79		1	3	7	5	2	18	0			
60–69		(+4) 2	(+3) 3	3	(−1) 1		9	−1	−9	9	+6
50–59		(+12) 3	(+2) 1	1			5	−2	−10	20	+14
40–49	(+18) 2						2	−3	−6	18	+18
f_x	2	6	8	18	11	5	50 N		−5 Σfd_y	75 Σfd^2_y	+53 ΣXY
d_x	−3	−2	−1	0	1	2					
fd_x	−6	−12	−8		11	10	−5 Σfd_x				
fd^2_x	18	24	8		11	20	81 Σfd^2_x				

The new value, ΣXY, is the algebraic sum of the individual XY values.

Note that frequencies within the mid-intervals have no XY value, because the deviation is zero.

Computation of r. — We are now ready to compute r, the coefficient of correlation by the product-moment method. The formula is as follows:

$$r = \frac{\frac{\Sigma XY}{N} - c_x c_y}{\sigma_x \sigma_y}$$

N = the number of cases.

ΣXY = the sum of the XY values.

c_x = (the correction) $\frac{\Sigma fd}{N}$ for the x series.

$c_y = \frac{\Sigma fd}{N}$ for the y series.

σ_x = the standard deviation of the x series, computed without multiplying by the size of the interval.

σ_y = the standard deviation of the y series, similarly computed.

Substituting numbers for symbols, we have

$$r = \frac{\frac{5}{5}\frac{3}{0} - (\frac{-5}{5}\frac{}{0} \times \frac{-5}{5}\frac{}{0})}{\sqrt{\frac{8}{5}\frac{1}{0} - (\frac{5}{5}\frac{}{0})^2} \times \sqrt{\frac{7}{5}\frac{5}{0} - (\frac{5}{5}\frac{}{0})^2}} = \frac{1.06 - .01}{\sqrt{1.62 - .10^2} \times \sqrt{1.50 - .10^2}}$$

$$r = \frac{1.05}{\sqrt{1.61} \times \sqrt{1.49}} = \frac{1.05}{1.26 \times 1.22} = \frac{1.05}{1.53}$$

$r = +.68$ (Ans.)

Outline of procedure. — The steps in computing r may be summarized as follows:

(*a*) Place before you the set of measures from which you wish to obtain the correlation. There must be two measures of each case.

(*b*) Inspect the two series carefully, to determine the range, and decide upon a suitable size of interval for each.

(*c*) Prepare a two-way diagram, with the x intervals at the top, reading from left to right, the y intervals at the left, reading from the bottom upward.

(*d*) Distribute the scores in their respective intervals, each tally mark representing the two measures of each case.

(*e*) Change the tally marks to numbers, and total each horizontal and vertical column. The grand total will be N, which should accord with the number of cases in the original data.

(*f*) Find the two mid-intervals.

(*g*) Mark in, for each series, the values of d, fd, and fd^2.

(*h*) Calculate the XY values, $f \times d \times d$. There will be an XY value for each frequency that does not occur within a mid-interval.

(*i*) Find the algebraic sum of each horizontal row of XY values. Place these sums, with appropriate sign, in the column marked XY.

(*j*) Find the algebraic sum of the XY column. This is ΣXY. Its sign, + or −, tells whether the correlation is positive or negative.

(*k*) Find the standard deviation (σ) of each series, without multiplying by the size of the interval.

(*l*) Follow the formula

$$r = \frac{\frac{\Sigma XY}{N} - c_x c_y}{\sigma_x \sigma_y}$$

(NOTE. — These computations are done algebraically. Be careful to watch the signs.)

ILLUSTRATIVE PROBLEM, *with solution.* — To find r:

	x										
	8–9	10–11	12–13	14–15	16–17	18–19	f_y	d_y	fd_y	fd^2_y	XY
80–89	(−6) 1	(−6) 2	2		(+6) 1	(+9) 1	7	+3	+21	63	+3
70–79	(−4) 1	(−6) 3	4	(+4) 2	(+4) 1	(+6) 1	12	+2	+24	48	+4
60–69		(−2) 2	5	(+1) 1	(+4) 2	(+3) 1	11	+1	+11	11	+6
y 50–59	1	4	6	2	2	1	16	0			
40–49	(+4) 2	(+3) 3	3	(−3) 3	(−4) 2		13	−1	−13	13	0
30–39	(+4) 1	(+4) 2	5	(−4) 2	(−4) 1	(−12) 2	13	−2	−26	52	−12
20–29	(+12) 2	(+3) 1					3	−3	−9	27	+15
f_x	8	17	25	10	9	6	75 N		+8 Σfd_y	214 Σfd^2_y	+16 ΣXY
d_x	−2	−1	0	+1	+2	+3					
fd_x	−16	−17		+10	+18	+18	+13 Σfd_x				
fd^2_x	32	17		10	36	54	149 Σfd^2_x				

$$r = \frac{\frac{\Sigma XY}{N} - c_x c_y}{\sigma_x \sigma_y} = \frac{\frac{16}{75} - (\frac{13}{75} \times \frac{8}{75})}{\sqrt{\frac{149}{75} - (\frac{13}{75})^2} \times \sqrt{\frac{214}{75} - (\frac{8}{75})^2}}$$

$$r = \frac{.21 - (.17 \times .10)}{\sqrt{1.98 - 17^2} \times \sqrt{2.85 - 10^2}} = \frac{.20}{\sqrt{1.96} \times \sqrt{2.84}} = \frac{.20}{1.40 \times 1.68}$$

$$r = \frac{.20}{2.35} = +.08 \text{ (Ans.)}$$

PROBLEMS TO SOLVE

1. Find the correlation (r) for the following distribution:

y \ x	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99	f_y
65–69						1	1		2
60–64					2	1			3
55–59			1	2	2				5
50–54		2	1	3	1	1			8
45–49	1		1	5	2	1	1	1	12
40–44		1	2	4	2		1		10
35–39		1	1	5					7
30–34				2	1				3
f_x	1	4	6	21	10	4	3	1	50

2. Find r:

y \ x	15–19	20–24	25–29	30–34	35–39	40–44
36–37				4	3	
34–35				5	2	1
32–33			4	8		
30–31		3	7	2	1	
28–29		4	5	2		
26–27	4	5				

3. Find r:

x	y	x	y	x	y	x	y	x	y
58	74	55	50	47	68	61	68	61	82
56	55	67	88	46	48	64	58	47	49
67	62	62	58	68	65	59	57	62	73
55	46	58	53	51	32	66	93	56	55
62	81	55	55	51	76	45	43	51	65
46	58	52	63	61	57	55	50	64	71
63	62	57	52	72	91	62	63	53	75
55	41	54	68	52	58	54	56	62	48
63	77	40	30	45	35	56	67	74	98
56	42	59	60	65	83	51	52	66	52
53	57	52	36	48	64	56	51	58	61
57	66	45	35	50	40	67	75	50	40
57	43	52	42	56	73	45	35	55	75
55	50	57	77	36	28	53	45	61	47
58	48	40	22	67	55	55	62	51	38

(Suggestion. — Use intervals of 5 for the x series, and intervals of 10 for the y series.)

4. Find r:

x	y	x	y	x	y	x	y	x	y
106	145	49	82	48	76	27	43	75	130
70	80	62	77	53	143	47	77	95	110
66	91	48	102	71	96	47	132	77	109
79	78	40	110	63	87	80	112	71	116
99	81	43	81	80	62	72	117	81	55
61	78	91	70	80	93	77	58	65	45
47	81	55	150	46	78	78	41	112	117
47	101	72	97	50	25	110	130	68	93
33	111	64	89	16	90	69	94	60	84
106	83	79	61	73	118	61	85	37	27
40	95	77	91	35	70	25	50	91	137
47	142	50	90	20	74	35	21	78	123
73	98	62	76	107	125	100	115	19	55
65	90	51	118	70	95	87	103	95	130
78	77	60	101	62	86	70	115	67	92
102	95	100	65	81	63	47	51	40	30

(Suggestion. — Use intervals of 15 for x, and intervals of 20 for y.)

5. Find r:

x	y	x	y	x	y	x	y	x	y
90	98	82	81	47	73	48	76	92	75
81	76	83	80	80	79	55	74	41	78
65	81	86	81	81	86	105	86	75	87
70	81	101	93	85	82	82	77	85	71
71	76	55	81	96	80	82	75	73	71
80	89	79	83	91	91	81	84	95	86
87	85	70	92	91	78	88	83	85	77
80	84	68	78	75	83	82	93	88	81
81	83	75	75	75	94	93	76	90	82
80	82	82	87	57	71	70	85	95	81
81	97	86	85	101	87	81	73	88	80
89	79	84	81	81	78	78	73	75	86
67	83	87	82	83	76	61	72	87	81
71	82	102	92	80	85	91	87	71	88
61	76	90	79	98	83	84	76	86	82
73	77	78	81	81	92	82	83	87	83
81	88	71	93	92	77	89	84	88	84
88	87	55	76	85	78	83	91	89	80

(SUGGESTION. — Use intervals of 10 for x, 5 for y.)

6. Find r:

x	y	x	y	x	y	x	y	x	y
7	90	3	61	6	65	4	51	4	52
4	54	6	81	4	54	4	88	4	53
5	81	4	53	6	87	4	50	4	59
4	61	5	62	3	53	3	55	5	75
4	68	5	85	2	55	6	66	2	57
4	69	4	63	5	64	4	72	5	53
5	49	6	63	5	89	3	40	2	41
3	48	4	66	4	65	4	76	4	74
3	47	3	77	3	46	3	34	2	37
4	41	4	62	4	64	4	67	4	78
6	98	3	63	3	67	6	71	6	51
4	55	6	85	3	69	6	55	6	58
5	61	4	51	4	52	4	86	4	45
5	83	3	51	2	68	3	57	3	59
4	62	5	63	4	71	5	51	5	57
6	62	5	87	3	41	2	40	2	51
4	67	4	64	5	65	4	73	5	55
3	75	6	67	3	33	4	31	1	47
5	47	4	65	4	66	4	77	4	75
4	45	4	63	3	38	3	35	2	33

7. Find r:

x	y	x	y	x	y	x	y	x	y
66	96	68	98	65	91	63	92	60	92
56	90	55	94	50	74	52	72	54	70
61	86	56	86	58	86	52	88	46	66
61	82	36	71	47	68	41	73	43	71
55	80	57	82	59	84	45	67	45	62
50	81	45	83	42	83	40	65	42	67
56	78	58	77	39	65	39	67	35	69
50	75	52	77	54	79	51	78	51	76
45	75	48	78	42	77	36	75	37	63
55	74	45	70	46	72	48	74	48	73
67	97	61	95	62	93	64	90	51	66
57	93	50	65	51	73	53	71	54	72
63	87	57	85	51	87	41	89	52	67
63	80	37	73	40	74	42	72	44	70
56	81	58	83	58	81	46	64	43	64
53	82	46	81	43	80	41	66	43	68
57	79	55	76	38	66	36	68	31	66
51	76	53	78	50	79	52	77	31	64
46	77	41	76	43	79	32	63	36	64
32	60	45	71	47	73	49	74	47	72

8. Find r:

x	y	x	y	x	y	x	y	x	y
110	81	103	65	94	66	93	67	90	69
101	69	106	83	91	83	94	58	94	75
81	61	97	63	91	68	93	68	92	57
101	78	102	77	93	70	92	70	104	71
85	64	87	62	89	60	101	72	104	73
90	74	92	72	90	70	102	77	94	72
100	79	95	79	87	58	97	78	76	54
95	65	98	66	96	76	91	63	86	67
81	56	85	55	95	70	99	73	93	61
96	73	98	72	84	59	79	57	79	53
90	65	91	67	92	68	108	77	91	56
83	64	94	66	106	76	94	65	80	52
96	61	82	60	92	67	91	78	81	53
96	56	88	61	94	71	93	74	90	71
86	63	103	79	104	75	83	54	94	72
91	73	91	71	103	76	103	78	87	68
96	68	86	59	88	57	85	65	89	67
99	69	97	67	90	67	92	62	85	52
97	71	89	58	76	55	94	60	93	53
82	57	95	74	97	72	72	46	77	51

9. Find r:

x	y	x	y	x	y	x	y	x	y	x	y
85	84	76	77	78	78	75	72	77	76	74	78
86	82	71	77	78	79	79	83	57	71	63	73
80	80	68	77	73	79	70	80	73	75	63	75
79	80	71	74	70	76	65	74	78	75	74	77
80	79	81	84	78	77	71	79	77	72	75	76
75	78	82	83	73	77	78	79	73	83	71	72
70	78	76	81	70	74	66	79	72	80	74	75
80	76	73	80	75	74	72	76	67	75	79	75
75	76	77	79	76	84	78	77	73	79	79	72
70	76	77	78	77	82	74	77	62	72	74	73
65	76	72	78	78	81	72	74	76	71	69	73
70	75	76	76	61	73	77	74	74	76	69	75
89	85	77	76	70	78	76	73	76	77	73	79
81	82	72	76	79	79	70	82	64	70	66	72
75	80	77	75	74	78	71	81	72	74	64	74
78	81	73	75	71	77	66	75	79	74	56	73
76	78	83	85	79	76	72	78	78	73	75	77
76	79	84	82	74	76	77	78	78	82	74	73
71	79	77	80	71	75	67	68	73	81	77	75
75	77	60	72	76	75	73	77	68	74	78	74

10. Find r:

x	y	x	y	x	y	x	y	x	y
60	75	91	71	81	66	81	71	91	60
100	70	85	68	75	60	85	55	75	50
63	68	80	60	72	76	84	69	83	75
85	76	82	67	100	63	75	50	80	55
90	65	110	55	50	65	81	60	92	67
70	65	76	73	84	60	75	72	120	70
60	60	92	60	95	55	62	67	101	62
70	50	83	68	73	77	110	63	100	65
78	70	100	66	100	55	90	70	71	66
110	60	90	55	64	69	65	63	75	55
75	75	100	50	93	68	80	55	82	60
83	60	80	70	60	45	110	70	93	75
80	50	90	50	95	56	70	55	70	60
80	55	70	45	97	78	85	48	100	67
60	65	61	66	75	55	72	67	95	62
65	45	105	53	95	50	83	50	91	66
93	60	70	50	100	55	90	55	86	65
105	57	85	50	75	46	110	57	90	60
100	45	90	45	70	50	80	45	80	55
80	65	85	60	100	60	90	55	110	48

11. Find r:

x	y	x	y	x	y	x	y	x	y
110	56	40	30	90	30	105	46	90	50
90	45	50	25	75	20	98	6	120	35
60	30	45	35	75	60	60	40	105	22
90	60	60	31	105	22	80	14	50	15
75	30	90	30	75	40	27	6	35	8
45	40	50	25	46	36	77	48	105	48
110	35	92	65	60	32	93	45	60	15
75	20	65	25	80	45	90	30	65	7
75	40	91	46	105	30	30	35	75	10
105	22	45	50	75	20	47	37	75	60
65	25	75	30	105	64	60	33	50	25
90	30	80	35	40	18	93	65	50	3
130	45	75	10	75	30	70	26	50	30
80	35	95	35	60	50	90	20	105	54
55	50	75	55	92	47	50	50	60	34
70	35	90	20	80	25	75	30	45	40
50	16	105	10	130	68	92	24	107	30
90	55	106	30	60	15	50	18	85	38
102	28	80	55	76	41	75	50	60	50
126	24	120	35	105	22	100	25	92	24
60	15	100	25	90	52	72	25	80	25
65	45	35	15	92	7	125	25	75	30
105	10	60	10	80	35	96	36	100	15
45	6	42	5	115	5	90	52	60	45
90	10	75	8	90	15	38	7	130	18
80	25	40	20	35	45	80	35	90	30

12. Find r:

x	y	x	y	x	y	x	y	x	y
90	75	92	77	80	75	51	66	50	71
90	70	91	73	95	73	81	72	74	72
80	65	82	67	70	65	72	67	61	64
80	64	71	63	73	61	60	64	50	60
57	31	26	33	12	32	45	62	59	61
70	55	60	55	62	57	64	59	15	34
50	55	52	57	54	59	55	55	17	36
40	55	48	57	49	55	31	59	30	43
80	50	70	51	72	53	25	44	51	52
60	54	62	52	64	50	66	52	57	54
50	50	52	54	55	51	57	52	59	50
40	54	42	54	48	53	46	53	50	41
30	54	39	51	57	43	59	41	55	44
80	49	75	48	63	46	68	48	67	48
50	45	52	47	54	49	56	47	58	45
58	49	56	47	54	45	41	44	44	42
40	46	49	45	35	49	37	47	27	46
79	43	76	41	75	43	40	35	42	37
65	41	68	43	68	43	67	42	37	35
87	38	77	35	67	37	51	35	53	37
91	76	93	78	13	33	52	67	55	74
90	74	99	71	80	71	72	73	75	73
81	66	83	68	71	66	60	65	50	65
70	64	72	62	72	60	61	63	51	64
58	33	11	31	40	64	55	64	58	63
71	56	61	56	63	58	65	55	17	32
51	56	53	58	55	59	58	56	39	41
49	56	47	56	30	55	33	56	35	40
89	53	71	52	75	51	27	41	57	53
61	53	63	51	65	51	56	54	58	50
51	51	52	53	56	54	58	53	57	53
41	53	45	51	49	50	48	52	53	42
38	52	56	44	58	42	57	43	54	43
70	46	61	45	65	47	69	49	47	44
51	46	53	48	55	48	57	46	59	46
57	48	55	46	53	46	43	42	49	43
45	47	47	46	36	46	25	49	15	47
78	44	77	40	76	43	41	36	36	39
66	40	69	44	65	41	26	37	55	39
75	38	64	36	69	35	52	36	54	38

REFERENCES ON THE PRODUCT-MOMENT METHOD

Brown & Thompson, *Essentials of Mental Measurement*, p. 107.
Chaddock, *Principles and Methods of Statistics*, p. 266.
Garrett, *Statistics in Psychology and Education*, p. 163.
Hines, *A Guide to Educational Measurements*, p. 34.
Holzinger, *Statistical Methods for Students in Education*, p. 149.
Kelley, *Statistical Method*, p. 153.
King, *Elements of Statistical Method*, p. 200.
Lincoln, *Beginnings in Educational Measurement*, p. 140.
Odell, *Educational Statistics*, p. 150.
Otis, *Statistical Method in Educational Measurement*, p. 186.
Rugg, *A Primer of Graphics and Statistics*, p. 99.
Thurstone, *The Fundamentals of Statistics*, p. 205.
Trabue, *Measuring Results in Education*, p. 392.

PART V

GRAPHIC METHODS

CHAPTER XXI

THE MEANING AND USE OF GRAPHIC METHODS

Explanation. — Graphic methods are ways of showing facts in the form of charts. Their usefulness is based on the fact that visual concepts are often easier to form than abstract concepts. This principle has been extensively applied in advertising, and has been found effective in statistical reports. There is an increasing tendency to use graphic methods, as may be seen by glancing through the pages of recent books and magazines dealing with numerical facts.

Although graphic methods may be used to produce striking effects, or to popularize facts which might otherwise be dry and uninteresting, perhaps the chief reason for their use in the study of statistical methods is to enable the reader to form a clearer notion of what statistical tabulations mean. Any person who knows how to make a frequency distribution, as shown in the earlier chapters of this book, ought to learn also how to chart a distribution in the form of a frequency curve. It will often happen that such a curve will give a new meaning to the distribution, and that the real nature of the group, and of individual differences within it, will be better understood. This is particularly true when it is desired to compare one group with another, or to show how a group stands in relation to a set of standards, or norms.

Graphic methods have come to be accepted as a part of statistical procedure. In recent years there has

been a movement to standardize the construction of charts, and to study the problem with a view to the adoption of the most effective and truthful methods.

Types of graphic methods. — The principal forms of graphic representation for statistical data are curves, frequency surfaces, bar diagrams, and individual frequency distributions. These methods will be illustrated in the concluding chapters of this book. There are, of course, many other graphic methods, including the use of simple areas, circles, organization charts, genealogical charts, maps and picture diagrams. Most of these are based on some form of statistical data, and a study of them will be found interesting and valuable. The references at the end of this chapter include the principal books dealing with the subject.

Methods of preparing charts. — The forms of charting illustrated in this book may be used for some immediate purpose, as the illustration of a report, demonstration, or lecture; or for the purposes of publication. In the former case the work may be done without much practice, and without special equipment, excepting, perhaps, cross-ruled paper. To be ready for publication, a chart should be drawn in India ink, on smooth white paper, in accordance with mechanical drawing procedure. Complete directions for this will be found in the eighth reference of the following list.

REFERENCES ON GRAPHIC METHODS

Alexander, Carter. *School Statistics and Publicity.* New York: Silver, Burdett & Co., 1919.

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King, W. I. *Elements of Statistical Method.*
McCall, W. A. *How to Measure in Education.*
Rugg, Harold. *A Primer of Graphics and Statistics.*
Secrist, Horace. *An Introduction to Statistical Methods.*
Williams, J. Harold. *Graphic Methods in Education.* Boston: Houghton Mifflin Co., 1924.

CHAPTER XXII

THE FREQUENCY CURVE

Explanation. — A frequency curve is a graphic illustration of a frequency distribution. It is based on two scales; the horizontal, which represents the intervals, and the vertical, which represents the frequencies.

The two scales. — The *vertical scale* is shown at the left of the chart, beginning with zero at the base line, and continuing upward to a point a little beyond the level of the highest frequency in the distribution. Convenient points are indicated, such as 10, 20, 30, etc., from which the horizontal *guide lines* are drawn through the chart. The spacing of these points depends upon the extent of the frequencies and the general nature of the data shown. The guide lines are useful in determining the points of the curve and in reading the completed chart.

The *horizontal scale* reads from left to right, and contains as many units as there are intervals to be shown. It will not begin at zero, unless the lowest interval happens to begin at that point. Numbers representing the intervals are placed under the base line, each at the bottom of a vertical line which extends to the top of the chart. These vertical lines intersect the horizontal guide lines, making the points necessary to the indication of the frequencies.

When the horizontal and vertical lines have been drawn and the frequency and interval scales numbered, the procedure is to make a point on each vertical line

the proper distance above the base line to indicate the frequency of the interval. When these points are indicated, they are connected with straight lines, and the chart is complete.

The form of the curve. — The frequency curve is a picture of a frequency distribution. It usually takes on the form of a mountain having one prominent peak. The peak, of course, is the mode, or the modal interval. A bimodal or multimodal distribution will have as many peaks as modes, each of which indicates a point of relatively high frequency.

Double and multiple frequency curves. — The frequency curve chart is not limited to a single distribution. Two or more curves may be drawn for comparative purposes, and doing so often brings striking results. This is particularly true in showing the results of a test or measurement which has been applied to two groups; or for dividing a group into two comparative groups, as is often done when measurements of boys and girls are compared. In such cases the number of intervals in the two distributions need not be the same, and of course the frequencies will usually be different; but there must be only one set of intervals, and there must be enough intervals and enough space in the vertical scale to provide completely for both distributions. It is also good practice, in drawing two or more curves on the same chart, to use a distinctive kind of line for each group. The commonest forms for this purpose are as follows:

Continuous line: ———————————

Dash line: — — — — — — — — — — — — — — —

Dot-dash line: — · — · — · — · — · — · —

There are many variations of these, as well as other types of line, and each chart-maker is privileged to use any style which may best represent his data. In some charts, colors may be used to good advantage. Obviously any comparison is most effective when sufficient contrast is shown.

Outline of procedure. — The steps in making a single frequency curve may be outlined as follows:

(*a*) Obtain the necessary data, in the form of a complete frequency distribution. (Any frequency distribution in this book may be used for practice.) Also obtain a suitable sheet of paper, which preferably should be cross-ruled.

(*b*) Draw a heavy horizontal line near the bottom of the sheet. This will be the base line.

(*c*) Lay off on the base line as many points, equally spaced, as there are intervals in the distribution. The spacing should be adjusted to the desired width of the chart.

(*d*) Draw a thin vertical line from each of these points on the base line, extending to the top of the chart.

(*e*) Look for the largest frequency in the distribution. Add enough to it to make the next round number. Let this number be the measure of the height of the chart.

(*f*) Draw a vertical line at the left of the chart, upward from the end of the base line and continuing to the top of the chart.

(*g*) Divide the vertical line into a number of convenient parts, such as twos, fives, or tens, with zero at the base line, and the "round number" chosen in (*e*) at the top.

(*h*) From each of the points on the vertical scale draw a thin horizontal line through the chart. These are the guide lines.

(NOTE: The number of guide lines will depend upon the nature of the distribution and the size of the chart. There is no universal rule about how many there should be. The more guide lines, the easier in making the curve; but too many of them may spoil the effect of the chart.)

(*i*) Using the horizontal lines as guides, make a point on each vertical line at the proper height to indicate the frequency of the interval.

(*j*) Connect the points to make a continuous curve. This will be made up of a series of straight lines between each pair of adjacent points.

(*k*) Label the horizontal scale according to the terms of the measurement; as feet, inches, pounds, years, I. Q., etc. Label the vertical scale "No. of cases," or "*f*."

On the next page is an illustrative problem with a completed frequency curve charting a distribution of scores obtained by sixth-grade pupils in an achievement test. Upon examination of the distribution it is found that the largest frequency is 48 so that 50 will be the measure of the height of the chart. On the page following this solution are four diagrams illustrating the steps taken in drawing such a chart.

ILLUSTRATIVE PROBLEM. — Chart the following distribution in the form of a frequency curve:

INT.	f
95–99	10
90–94	30
85–89	42
80–84	48
75–79	34
70–74	30
65–69	12
60–64	8
N	214

(Representing the scores obtained by a group of sixth grade pupils in an achievement test.)

Solution:

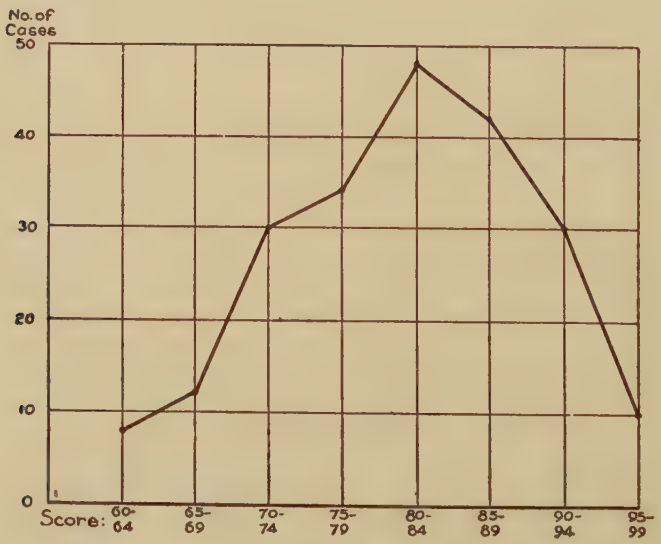

FIG. 1. DISTRIBUTION OF SCORES OBTAINED BY A GROUP OF SIXTH-GRADE PUPILS IN AN ACHIEVEMENT TEST

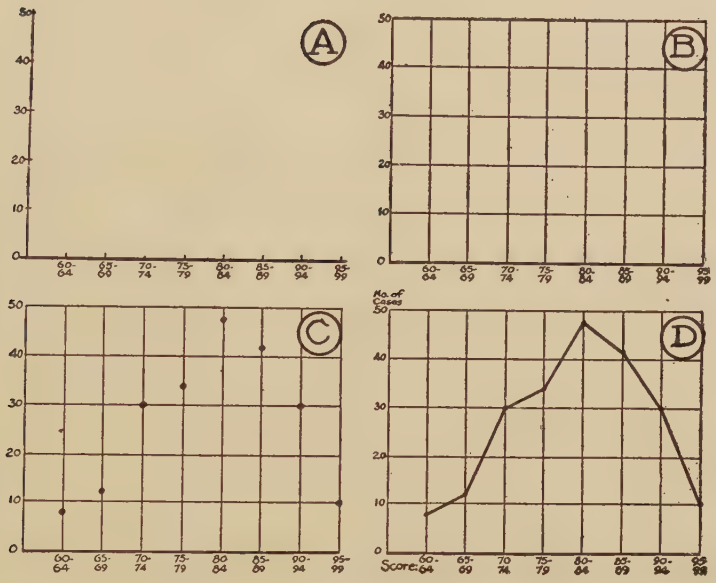

FIG. 2. STEPS TAKEN IN DRAWING THE CHART SHOWN AS FIG. 1

A. Showing vertical and horizontal scales marked off.

B. Showing vertical lines and horizontal guide lines, drawn from points indicated on the two scales.

C. Showing location of frequency points, each representing the frequency of its interval. The frequencies, reading from left to right, are 8, 12, 30, 34, 48, 42, 30, 10.

D. Showing the finished chart, completed by connecting the frequency points with heavy straight lines.

ILLUSTRATIVE PROBLEM. — Chart the following data, representing the standing of the Washington and Jefferson schools in a test of reading:

SCORE	WASHINGTON	JEFFERSON
80–84	2	5
75–79	8	12
70–74	10	15
65–69	17	19
60–64	20	25
55–59	24	27
50–54	20	25
45–49	15	18
40–44	18	15
35–39	15	10
30–34	8	0
25–29	3	2
TOTAL	160	173

Solution:

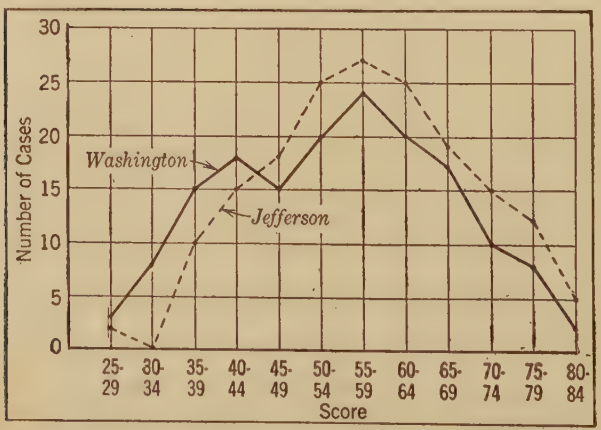

FIG. 3. DISTRIBUTION OF SCORES IN READING, WASHINGTON AND JEFFERSON SCHOOLS

PROBLEMS TO SOLVE

Chart each of the following distributions in the form of a frequency curve:

1.

INT.	*f*
90–99	2
80–89	5
70–79	8
60–69	12
50–59	18
40–49	24
30–39	20
20–29	15
10–19	8
0–9	3

2.

INT.	*f*
95–99	3
90–94	0
85–89	4
80–84	5
75–79	8
70–74	14
65–69	20
60–64	28
55–59	35
50–54	44
45–49	40
40–44	18
35–39	10

3.

INT.	*f*
60–61	4
58–59	8
56–57	10
54–55	12
52–53	18
50–51	27
48–49	40
46–47	62
44–45	56
42–43	45
40–41	30
38–39	22
36–37	15

4.

INT.	*f*
16	10
15	12
14	15
13	24
12	32
11	30
10	28
9	30
8	24
7	18
6	14
5	7

5.

INT.	*f*
75–79	2
70–74	0
65–69	0
60–64	4
55–59	8
50–54	10
45–49	10
40–44	7
35–39	12
30–34	9
25–29	4
20–24	0
15–19	2

6.

INT.	*f*
60–62	2
57–59	4
54–56	0
51–53	5
48–50	7
45–47	15
42–44	22
39–41	10
36–38	7
33–35	3
30–32	8
27–29	12
24–26	16
21–23	4

NOTE: *The remaining problems (7 to 12) are not properly frequency curves, but are made in a similar manner by constructing a scale on which the measurements may be plotted.*

7. Following are the average weights, in kilograms, of a group of boys and girls. Show the data in the form of a double curve:

AGE	BOYS	GIRLS
18	61	52
17	57	52
16	53	50
15	47	48
14	42	44
13	38.5	38
12	34	34
11	31	30
10	28	27
9	26	25

8. Make into a double curve:

NORMS OF VITAL CAPACITY, IN CUBIC CENTIMETERS

AGE:	6	7	8	9	10	11	12	13	14
BOYS:	1023	1168	1316	1469	1603	1732	1883	2108	2395
GIRLS:	950	1061	1165	1286	1409	1526	1664	1827	2014

9. Make into a double curve:

AVERAGE DAILY ATTENDANCE OF JUNIOR HIGH SCHOOL PUPILS

MONTH:	EIGHTH GRADE:	NINTH GRADE:
September	92.7	95.3
October	96.8	96.7
November	97.4	97.2
December	95.8	93.8
January	92.9	94.7
February	90.6	90.9
March	88.5	87.3
April	87.2	85.2
May	86.4	84.7

10. Make into a double curve:

SCORES OF BOYS IN TAPPING TEST

AGE:	8	9	10	11	12	13	14	15	16
RIGHT HAND:	147	151	161	169	170	184	184	191	196
LEFT HAND:	117	127	132	141	145	156	155	169	170

11. Make into a double curve:

SCORES IN DIGIT-MEMORY TEST OF BRIGHT AND DULL PUPILS

AGE:	9	10	11	12	13	14	15
BRIGHT:	47.8	54.4	59.0	62.6	70.4	68.9	68.9
DULL:	39.7	42.7	48.6	52.2	64.3	62.6	62.4

12. Make into a multiple-curve chart:

SCORES OBTAINED BY THREE SCHOOLS IN SILENT READING TEST

GRADE:	II	III	IV	V	VI	VII	VIII
LINCOLN:	28	30	32	35	38	42	43
FRANKLIN:	25	28	29	35	36	40	42
GRANT:	20	24	25	30	34	34	35

CHAPTER XXIII

THE FREQUENCY SURFACE

Explanation. — The frequency surface, or polygon, as it is sometimes called, is a modified form of frquency curve. It is usually based on intervals and frequencies, as in the case of the curve, but instead of indicating each frequency with a point, its *level* is indicated by means of a horizontal line drawn through the space representing the interval, and at a height above the base line which corresponds to the frequency. The straight horizontal lines are then connected with vertical lines, giving the general effect of a stairway, usually with the highest step near the center, and leading downward toward the right. The degree of symmetry obtained will, of course, vary with the nature of the distribution.

The frequency surface chart is sometimes more easily understood than the curve type, although it is not considered as desirable for many types of distribution. The usefulness of any form of chart, however, is so dependent upon the nature of the data and the practical purpose for which the chart is made, that the choice is largely an individual matter.

The frequency surface chart may be used for comparing two or more distributions, by the use of different types of line, as explained in Chapter 22. In most cases the frequency curve will be found preferable for such comparisons.

Outline of procedure. — The steps in laying out and drawing a frequency surface chart are as follows:

(*a*) Obtain the data, in the form of a frequency distribution.

(*b*) Draw the base line of the chart. Its length will depend upon the size of chart desired.

(*c*) Lay off, on the base line, as many *spaces* (not points) as there are intervals in the distribution. The base line should project slightly at each end.

(*d*) Indicate the intervals, each in the center of the proper space, below the base line.

(*e*) Draw a series of thin vertical lines from the base line to the top of the chart, marking off the interval spaces in the form of vertical columns.

(*f*) Lay off the vertical scale at the left of the chart, according to the largest frequency to be shown.

(*g*) Draw horizontal guide lines through the chart from appropriate points on the vertical scale. Number these at the left, in terms of the frequencies they represent.

(*h*) Draw a short horizontal line across each of the vertical columns, at a height which corresponds to the frequency of the interval.

(*i*) Connect the ends of the horizontal lines with vertical lines of the same thickness, drawn along the thin vertical lines required for step (*e*). This completes the chart.

ILLUSTRATIVE PROBLEM. — Make a frequency surface chart from the following data:

SCORES OBTAINED IN STANFORD ACHIEVEMENT TEST BY EIGHTH-GRADE PUPILS IN MONROVIA, CALIFORNIA

SCORE	f
95–99	1
90–94	8
85–89	15
80–84	17
75–79	24
70–74	20
65–69	21
60–64	9
55–59	5
50–54	4
TOTAL	124

Solution:

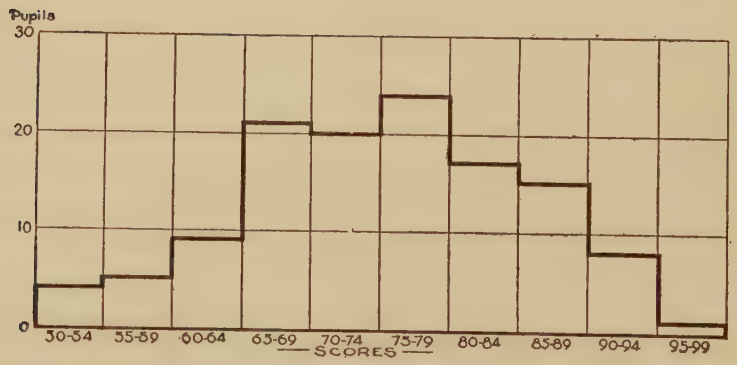

FIG. 4. SCORES OBTAINED IN STANFORD ACHIEVEMENT TEST BY EIGHTH-GRADE PUPILS IN MONROVIA, CALIFORNIA

PROBLEMS TO SOLVE

Chart each of the following distributions in the form of a frequency surface:

1. INTELLIGENCE QUOTIENTS OF 100 SPECIAL-CLASS PUPILS

I. Q.:	40–49	50–59	60–69	70–79	80–89	90–99	100–109
f:	8	12	25	28	16	7	4

2. AGES OF A GROUP OF SALESMEN

AGE:	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59
f:	16	29	45	53	20	14	5	3

3. HEIGHT, IN CENTIMETERS, OF A GROUP OF TEN-YEAR-OLD BOYS

HEIGHT:	118–119	120–121	122–123	124–125	126–127	128–129	130–131	132–133	134–135	136–137
f:	1	0	5	4	10	25	63	30	16	9

4. DISTRIBUTION OF WEEKLY WAGES IN A DEPARTMENT STORE

WAGES:	5–9	10–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64	65–69
f:	3	20	82	75	28	22	20	15	10	8	5	2	2

5. EXPERIENCE, IN YEARS OF A GROUP OF SALESMEN

YEARS:	0–1	2–3	4–5	6–7	8–9	10–11	12–13	14–15	16–17	18–19	20– and over
f:	18	40	35	20	18	10	7	3	0	0	4

CHAPTER XXIV

THE BAR DIAGRAM

Explanation. — The bar diagram, like the frequency surface, is a modified form of frequency curve. Instead of using points or levels, the frequency of each interval is indicated by a column extending from the base line to a proportionate height.

Bar diagrams are of two types — *vertical* and *horizontal.* In general construction they are the same, the difference being that the former consists of vertical bars arising from a horizontal base line, as in the case of the frequency surface; while the latter consists of horizontal bars extending from a vertical base line. The manner of construction is otherwise identical. The vertical bar diagram has the general appearance of a frequency surface, but it will be observed that the columns are separated, so the continuous " surface " effect is not obtained. However, a bar diagram may easily be converted into a frequency surface, and *vice versa.*

Outline of procedure for vertical bar diagram. — The procedure is similar to that given in Chapter 23. The steps are as follows:

(*a*) Obtain the data, in the form of a frequency distribution.

(*b*) Draw the base line.

(*c*) Lay off, on the base line, a number of *slightly separated* spaces, according to the number of intervals in the distribution.

(*d*) Indicate the intervals, each in its own space.

(*e*) Lay off and number the vertical scale.

(*f*) Draw the horizontal guide lines.

(*g*) Construct, on each interval space, a vertical column of the correct height to indicate the frequency.

(*h*) Fill in the vertical columns so that each will stand out in contrast to the surrounding part of the chart.

ILLUSTRATIVE PROBLEM. — Make a vertical bar diagram of the following data:

READING SCORES OBTAINED BY A GROUP OF EIGHTH-GRADE PUPILS IN STANFORD ACHIEVEMENT TEST

SCORE	*f*
250–259	1
240–249	2
230–239	5
220–229	8
210–219	9
200–209	10
190–199	13
180–189	13
170–179	11
160–169	7
150–159	4
140–149	3

Solution:

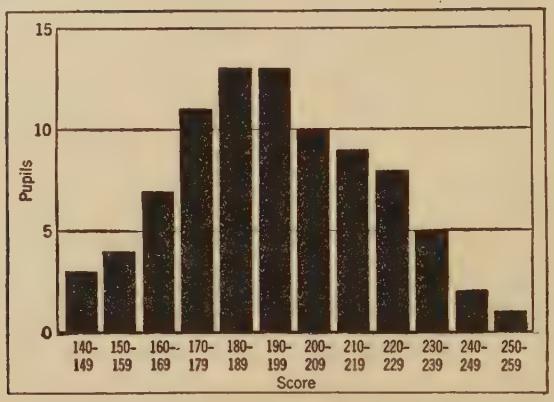

FIG. 5. READING SCORES OBTAINED BY A GROUP OF EIGHTH-GRADE PUPILS IN STANFORD ACHIEVEMENT TEST

Outline of procedure for horizontal bar diagram. — The procedure is similar to that just described for vertical bars. The steps are as follows:

(*a*) Obtain the data, in the form of a frequency distribution.

(*b*) Draw the base line *vertically*, at the left.

(*c*) Lay off the necessary spaces on the base line.

(*d*) Indicate the intervals, to the left of the base line.

(*e*) Lay off the frequency scale *horizontally*.

(*f*) Draw the vertical guide lines.

(*g*) Construct a horizontal bar for each interval, each of appropriate length to represent the frequency.

(*h*) Fill in the bars so they will stand out prominently.

ILLUSTRATIVE PROBLEM. — Make a horizontal bar chart of the following data:

AGES OF A GROUP OF DELINQUENT BOYS

AGE:	9	10	11	12	13	14	15	16	17	18	19
f:	3	5	8	17	25	38	30	24	12	6	2

Solution:

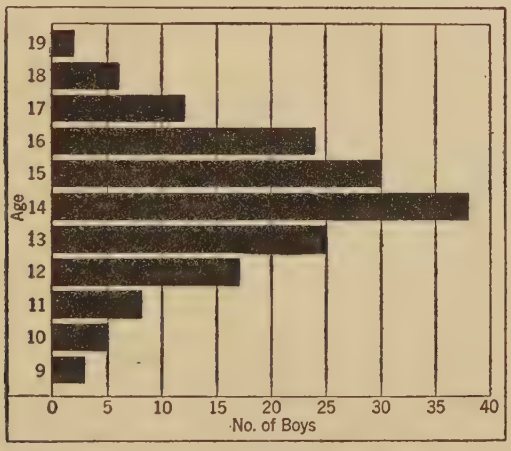

FIG. 6. AGES OF A GROUP OF DELINQUENT BOYS

PROBLEMS TO SOLVE

Chart in the form of a bar diagram, choosing either the vertical or horizontal type, depending upon its effectiveness for each problem:

1. DISTRIBUTION OF TEACHERS' SALARIES IN A WESTERN CITY

SALARY:	$1400	1500	1600	1700	1800	1900	2000	2100	2200
f:	12	18	27	30	22	15	8	5	3

2. AGES OF A GROUP OF TENTH-GRADE BOYS

AGE	f
18.0–18.4	1
17.5–17.9	4
17.0–17.4	4
16.5–16.9	9
16.0–16.4	15
15.5–15.9	19
15.0–15.4	28
14.5–14.9	32
14.0–14.4	22
13.5–13.9	4
13.0–13.4	2

3. INTELLIGENCE QUOTIENTS OF A GROUP OF GIFTED PUPILS

I. Q.:	120–129	130–139	140–149	150–159	160–169	170–179	180–189	190–199
f:	5	12	27	20	10	5	2	1

4. SIZE OF ELEMENTARY SCHOOLS, BY NUMBER OF CLASSROOMS

NO. OF ROOMS:	4–5	6–7	8–9	10–11	12–13	14–15	16–17	18–19	20–21
f:	12	15	23	47	52	63	40	15	12

5. HANDWRITING QUALITY SCORES OF FOURTH-GRADE PUPILS

QUALITY:	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59
f:	15	18	36	31	39	10	10	4

CHAPTER XXV

THE INDIVIDUAL FREQUENCY DISTRIBUTION

Explanation. — The individual frequency distribution is characterized by the representation of every individual case by a spot, a circle, a block, a line, or some other unit. Spots and blocks are the most common forms used.

ILLUSTRATIVE PROBLEM. — Arrange in the form of an individual frequency distribution:

AGES OF A GROUP OF PRIMARY SCHOOL CHILDREN

AGES:	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	10.5	11.0
f:	1	2	4	5	7	8	8	7	6	6	5	4	2

Solution:

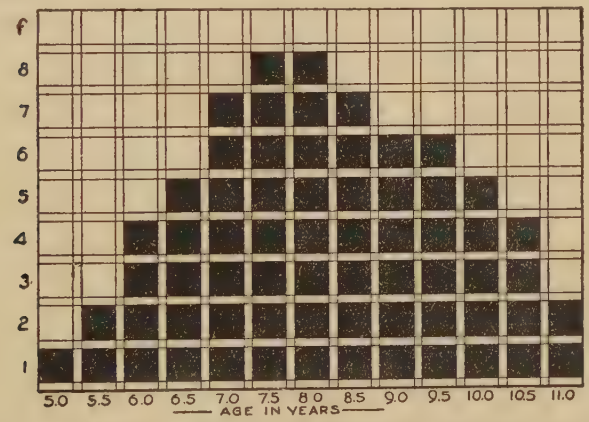

FIG. 7. AGES OF A GROUP OF PRIMARY SCHOOL CHILDREN

The two-way distribution. — One of the most useful forms of individual frequency chart is the two-way distribution, or " scatter diagram " (Fig. 8).

ILLUSTRATIVE PROBLEM. — Arrange in a two-way frequency distribution, or "scatter diagram":

SCORES OF SIXTY PUPILS IN READING AND SPELLING

RD.	SP.	RD.	SP.	RD.	SP.	RD.	SP.	RD.	SP.
30	60	5	10	27	60	28	40	45	100
10	40	10	15	25	40	30	50	40	75
50	100	15	30	40	40	22	50	33	70
45	80	28	40	37	70	35	85	33	60
25	55	17	25	15	18	20	35	32	65
45	85	54	110	22	65	28	70	37	80
29	80	28	66	19	100	35	60	5	20
43	60	37	65	30	70	15	45	42	100
30	89	20	25	39	90	31	40	40	60
27	46	33	46	15	80	29	49	3	0
15	35	25	70	40	85	28	61	20	40
25	30	32	75	12	65	18	70	12	30

Solution:

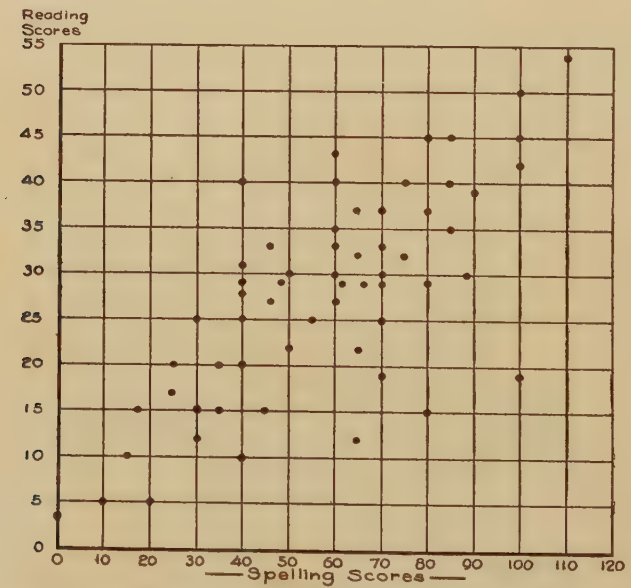

FIG. 8. SCORES OF SIXTY PUPILS IN READING AND SPELLING

Each vertical and horizontal line represents the beginning of an interval. Each spot represents one case, and is located in its proper place on both scales. Note the first pair of scores, Reading 30, Spelling 60. The spot is located at the intersection of the 30 horizontal and 60 vertical lines. Note the sixth pair, Reading 45, Spelling 85. The spot occurs on the 45 horizontal line and *half way* between the 80 and 90 vertical lines.

PROBLEMS TO SOLVE

1. Arrange as an individual frequency distribution:

GRADE DISTRIBUTION OF NEGRO CHILDREN IN AN ELEMENTARY SCHOOL

GRADE:	KINDERGARTEN	I	II	III	IV	V	VI	VII	VIII
f:	5	7	8	12	15	10	7	3	2

2. Arrange as an individual frequency distribution:

INTELLIGENCE QUOTIENTS OF A GROUP OF JUVENILE-COURT DETENTION CASES

I. Q.	*f*
120–129	3
110–119	7
100–109	10
90–99	15
80–89	16
70–79	10
60–69	4
50–59	2

3. Arrange as an individual frequency distribution:

SIZE OF A GROUP OF ELEMENTARY SCHOOLS

NO. OF ROOMS:	3	4	5	6	7	8	9	10	11	12
NO. OF BUILDINGS:	2	2	4	8	12	16	24	20	15	8

4. Arrange as a "scatter diagram":

HEIGHT AND WEIGHT OF A GROUP OF CHILDREN

HT.	WT.	HT.	WT.	HT.	WT.	HT.	WT.
50	60	70	105	40	45	35	40
50	40	45	40	55	85	40	50
30	25	35	65	60	90	60	120
50	85	65	50	43	62	50	83
60	120	60	125	55	90	47	85

(NOTE. — In choosing appropriate intervals the procedure is the same as for the distributions described in Chapter 20.)

5. Arrange as a "scatter diagram" the first illustrative problem in Chapter 20.

APPENDIX

TABLES TO FACILITATE THE CALCULATION OF RANK-DIFFERENCE CORRELATION

Table I. Values of $N(N^2-1)$.

When N is	$N(N^2-1)$ will be
5	120
6	210
7	336
8	504
9	720
10	990
11	1320
12	1716
13	2184
14	2730
15	3360
16	4080
17	4896
18	5814
19	6840
20	7980
21	9240
22	10626
23	12144
24	13800
25	15600
26	17550
27	19656
28	21924
29	24360
30	26970
31	29760
32	32736
33	33594
34	39270
35	42840
36	46620
37	50579
38	54834
39	59280
40	63960
41	68880
42	74046

Table II. Squares of Fractional and Large Differences.

When D is	D^2 will be
1.5	2.25
2.5	6.25
3.5	12.25
4.5	20.25
5.5	30.25
6.5	42.25
7.5	56.25
8.5	72.25
9.5	90.25
10.5	110.25
11.5	132.25
12.5	156.25
13.0	169.00
13.5	182.25
14.0	196.00
14.5	210.25
15.0	225.00
15.5	240.25
16.0	256.00
16.5	272.25
17.0	289.00
17.5	306.25
18.0	324.00
18.5	342.25
19.0	361.00
19.5	380.25
20.0	400.00
20.5	420.25
21.0	441.00
21.5	462.25
22.0	484.00
22.5	506.25
23.0	529.00
23.5	552.25
24.0	576.00
24.5	600.25
25.0	625.00

TABLE OF SQUARE ROOTS. 1.01 to 4.00

No.	Sq. r.	No.	Sq. r.	No.	Sq. r.	No.	Sq. r.	No.	Sq. r.	No.	Sq. r.
1.01	1.005	1.51	1.228	2.01	1.417	2.51	1.584	3.01	1.734	3.51	1.873
1.02	1.010	1.52	1.232	2.02	1.421	2.52	1.587	3.02	1.737	3.52	1.876
1.03	1.014	1.53	1.236	2.03	1.424	2.53	1.590	3.03	1.740	3.53	1.878
1.04	1.019	1.54	1.241	2.04	1.428	2.54	1.593	3.04	1.743	3.54	1.881
1.05	1.024	1.55	1.245	2.05	1.431	2.55	1.596	3.05	1.746	3.55	1.884
1.06	1.029	1.56	1.249	2.06	1.435	2.56	1.600	3.06	1.749	3.56	1.886
1.07	1.034	1.57	1.253	2.07	1.438	2.57	1.603	3.07	1.752	3.57	1.889
1.08	1.039	1.58	1.257	2.08	1.442	2.58	1.606	3.08	1.755	3.58	1.892
1.09	1.044	1.59	1.261	2.09	1.445	2.59	1.609	3.09	1.757	3.59	1.894
1.10	1.048	1.60	1.264	2.10	1.449	2.60	1.612	3.10	1.760	3.60	1.897
1.11	1.053	1.61	1.268	2.11	1.452	2.61	1.615	3.11	1.763	3.61	1.900
1.12	1.058	1.62	1.272	2.12	1.456	2.62	1.618	3.12	1.766	3.62	1.902
1.13	1.063	1.63	1.276	2.13	1.459	2.63	1.621	3.13	1.769	3.63	1.905
1.14	1.067	1.64	1.280	2.14	1.462	2.64	1.624	3.14	1.772	3.64	1.907
1.15	1.072	1.65	1.284	2.15	1.466	2.65	1.627	3.15	1.774	3.65	1.910
1.16	1.077	1.66	1.288	2.16	1.469	2.66	1.631	3.16	1.777	3.66	1.913
1.17	1.081	1.67	1.292	2.17	1.473	2.67	1.634	3.17	1.780	3.67	1.915
1.18	1.086	1.68	1.296	2.18	1.476	2.68	1.637	3.18	1.783	3.68	1.918
1.19	1.090	1.69	1.300	2.19	1.479	2.69	1.640	3.19	1.786	3.69	1.920
1.20	1.095	1.70	1.303	2.20	1.483	2.70	1.643	3.20	1.788	3.70	1.923
1.21	1.100	1.71	1.307	2.21	1.486	2.71	1.646	3.21	1.791	3.71	1.926
1.22	1.104	1.72	1.311	2.22	1.490	2.72	1.649	3.22	1.794	3.72	1.928
1.23	1.109	1.73	1.315	2.23	1.493	2.73	1.652	3.23	1.797	3.73	1.931
1.24	1.113	1.74	1.319	2.24	1.496	2.74	1.655	3.24	1.800	3.74	1.933
1.25	1.118	1.75	1.322	2.25	1.500	2.75	1.658	3.25	1.802	3.75	1.936
1.26	1.122	1.76	1.326	2.26	1.503	2.76	1.661	3.26	1.805	3.76	1.939
1.27	1.126	1.77	1.330	2.27	1.506	2.77	1.664	3.27	1.808	3.77	1.941
1.28	1.131	1.78	1.334	2.28	1.510	2.78	1.667	3.28	1.811	3.78	1.944
1.29	1.135	1.79	1.337	2.29	1.513	2.79	1.670	3.29	1.813	3.79	1.946
1.30	1.140	1.80	1.341	2.30	1.516	2.80	1.673	3.30	1.816	3.80	1.949
1.31	1.144	1.81	1.345	2.31	1.519	2.81	1.676	3.31	1.819	3.81	1.951
1.32	1.148	1.82	1.349	2.32	1.523	2.82	1.679	3.32	1.822	3.82	1.954
1.33	1.153	1.83	1.352	2.33	1.526	2.83	1.682	3.33	1.824	3.83	1.957
1.34	1.157	1.84	1.356	2.34	1.529	2.84	1.685	3.34	1.827	3.84	1.959
1.35	1.161	1.85	1.360	2.35	1.533	2.85	1.688	3.35	1.830	3.85	1.962
1.36	1.166	1.86	1.363	2.36	1.536	2.86	1.691	3.36	1.833	3.86	1.964
1.37	1.170	1.87	1.367	2.37	1.539	2.87	1.694	3.37	1.835	3.87	1.967
1.38	1.174	1.88	1.371	2.38	1.542	2.88	1.697	3.38	1.838	3.88	1.969
1.39	1.179	1.89	1.374	2.39	1.546	2.89	1.700	3.39	1.841	3.89	1.972
1.40	1.183	1.90	1.378	2.40	1.549	2.90	1.702	3.40	1.843	3.90	1.974
1.41	1.187	1.91	1.382	2.41	1.552	2.91	1.705	3.41	1.846	3.91	1.977
1.42	1.191	1.92	1.385	2.42	1.555	2.92	1.708	3.42	1.849	3.92	1.979
1.43	1.195	1.93	1.389	2.43	1.558	2.93	1.711	3.43	1.852	3.93	1.982
1.44	1.200	1.94	1.392	2.44	1.562	2.94	1.714	3.44	1.854	3.94	1.984
1.45	1.204	1.95	1.396	2.45	1.565	2.95	1.717	3.45	1.857	3.95	1.987
1.46	1.208	1.96	1.400	2.46	1.568	2.96	1.720	3.46	1.860	3.96	1.989
1.47	1.212	1.97	1.403	2.47	1.571	2.97	1.723	3.47	1.862	3.97	1.992
1.48	1.216	1.98	1.407	2.48	1.574	2.98	1.726	3.48	1.865	3.98	1.994
1.49	1.220	1.99	1.410	2.49	1.578	2.99	1.729	3.49	1.868	3.99	1.997
1.50	1.224	2.00	1.414	2.50	1.581	3.00	1.732	3.50	1.870	4.00	2.000

TABLE OF SQUARE ROOTS. 4.01 to 7.00

No.	Sq. r.	No.	Sq. r.	No.	Sq. r.	No.	Sq. r.	No.	Sq. r.	No.	Sq. r.
4.01	2.002	4.51	2.123	5.01	2.238	5.51	2.347	6.01	2.451	6.51	2.551
4.02	2.005	4.52	2.126	5.02	2.240	5.52	2.349	6.02	2.453	6.52	2.553
4.03	2.007	4.53	2.128	5.03	2.242	5.53	2.351	6.03	2.455	6.53	2.555
4.04	2.010	4.54	2.130	5.04	2.245	5.54	2.353	6.04	2.457	6.54	2.557
4.05	2.012	4.55	2.133	5.05	2.247	5.55	2.355	6.05	2.459	6.55	2.559
4.06	2.014	4.56	2.135	5.06	2.249	5.56	2.358	6.06	2.461	6.56	2.561
4.07	2.017	4.57	2.137	5.07	2.251	5.57	2.360	6.07	2.463	6.57	2.563
4.08	2.019	4.58	2.140	5.08	2.253	5.58	2.362	6.08	2.465	6.58	2.565
4.09	2.022	4.59	2.142	5.09	2.256	5.59	2.364	6.09	2.467	6.59	2.567
4.10	2.024	4.60	2.144	5.10	2.258	5.60	2.366	6.10	2.469	6.60	2.569
4.11	2.027	4.61	2.147	5.11	2.260	5.61	2.368	6.11	2.471	6.61	2.571
4.12	2.029	4.62	2.149	5.12	2.262	5.62	2.370	6.12	2.473	6.62	2.572
4.13	2.032	4.63	2.151	5.13	2.265	5.63	2.372	6.13	2.475	6.63	2.574
4.14	2.034	4.64	2.154	5.14	2.267	5.64	2.374	6.14	2.477	6.64	2.576
4.15	2.037	4.65	2.156	5.15	2.269	5.65	2.377	6.15	2.479	6.65	2.578
4.16	2.039	4.66	2.158	5.16	2.271	5.66	2.379	6.16	2.481	6.66	2.580
4.17	2.042	4.67	2.161	5.17	2.273	5.67	2.381	6.17	2.483	6.67	2.582
4.18	2.044	4.68	2.163	5.18	2.276	5.68	2.383	6.18	2.486	6.68	2.584
4.19	2.046	4.69	2.165	5.19	2.278	5.69	2.385	6.19	2.488	6.69	2.586
4.20	2.049	4.70	2.167	5.20	2.280	5.70	2.387	6.20	2.490	6.70	2.588
4.21	2.051	4.71	2.170	5.21	2.282	5.71	2.389	6.21	2.492	6.71	2.590
4.22	2.054	4.72	2.172	5.22	2.284	5.72	2.391	6.22	2.494	6.72	2.592
4.23	2.056	4.73	2.174	5.23	2.286	5.73	2.393	6.23	2.496	6.73	2.594
4.24	2.059	4.74	2.177	5.24	2.289	5.74	2.395	6.24	2.498	6.74	2.596
4.25	2.061	4.75	2.179	5.25	2.291	5.75	2.397	6.25	2.500	6.75	2.598
4.26	2.064	4.76	2.181	5.26	2.293	5.76	2.400	6.26	2.502	6.76	2.600
4.27	2.066	4.77	2.184	5.27	2.295	5.77	2.402	6.27	2.504	6.77	2.601
4.28	2.068	4.78	2.186	5.28	2.297	5.78	2.404	6.28	2.506	6.78	2.603
4.29	2.071	4.79	2.188	5.29	2.300	5.79	2.406	6.29	2.508	6.79	2.605
4.30	2.073	4.80	2.190	5.30	2.302	5.80	2.408	6.30	2.510	6.80	2.607
4.31	2.076	4.81	2.193	5.31	2.304	5.81	2.410	6.31	2.512	6.81	2.609
4.32	2.078	4.82	2.195	5.32	2.306	5.82	2.412	6.32	2.514	6.82	2.611
4.33	2.080	4.83	2.197	5.33	2.308	5.83	2.414	6.33	2.515	6.83	2.613
4.34	2.083	4.84	2.200	5.34	2.310	5.84	2.416	6.34	2.517	6.84	2.615
4.35	2.085	4.85	2.202	5.35	2.313	5.85	2.418	6.35	2.519	6.85	2.617
4.36	2.088	4.86	2.204	5.36	2.315	5.86	2.420	6.36	2.521	6.86	2.619
4.37	2.090	4.87	2.206	5.37	2.317	5.87	2.422	6.37	2.523	6.87	2.621
4.38	2.092	4.88	2.209	5.38	2.319	5.88	2.424	6.38	2.525	6.88	2.623
4.39	2.095	4.89	2.211	5.39	2.321	5.89	2.426	6.39	2.527	6.89	2.624
4.40	2.097	4.90	2.213	5.40	2.323	5.90	2.429	6.40	2.529	6.90	2.626
4.41	2.100	4.91	2.215	5.41	2.325	5.91	2.431	6.41	2.531	6.91	2.628
4.42	2.102	4.92	2.218	5.42	2.328	5.92	2.433	6.42	2.533	6.92	2.630
4.43	2.104	4.93	2.220	5.43	2.330	5.93	2.435	6.43	2.535	6.93	2.622
4.44	2.107	4.94	2.222	5.44	2.332	5.94	2.437	6.44	2.537	6.94	2.634
4.45	2.109	4.95	2.224	5.45	2.334	5.95	2.439	6.45	2.539	6.95	2.636
4.46	2.111	4.96	2.227	5.46	2.336	5.96	2.441	6.46	2.541	6.96	2.638
4.47	2.114	4.97	2.229	5.47	2.338	5.97	2.443	6.47	2.543	6.97	2.640
4.48	2.116	4.98	2.231	5.48	2.340	5.98	2.445	6.48	2.545	6.98	2.642
4.49	2.119	4.99	2.233	5.49	2.343	5.99	2.447	6.49	2.547	6.99	2.643
4.50	2.121	5.00	2.236	5.50	2.345	6.00	2.449	6.50	2.549	7.00	2.645

TABLE OF SQUARE ROOTS. 7.01 to 10.00

No.	Sq. r.	No.	Sq. r.	No.	Sq. r.	No.	Sq. r.	No.	Sq. r.	No.	Sq. r.
7.01	2.647	7.51	2.740	8.01	2.830	8.51	2.917	9.01	3.001	9.51	3.083
7.02	2.649	7.52	2.742	8.02	2.831	8.52	2.918	9.02	3.003	9.52	3.085
7.03	2.651	7.53	2.744	8.03	2.833	8.53	2.920	9.03	3.005	9.53	3.087
7.04	2.653	7.54	2.745	8.04	2.835	8.54	2.922	9.04	3.006	9.54	3.088
7.05	2.655	7.55	2.747	8.05	2.837	8.55	2.924	9.05	3.008	9.55	3.090
7.06	2.657	7.56	2.749	8.06	2.839	8.56	2.925	9.06	3.009	9.56	3.091
7.07	2.658	7.57	2.751	8.07	2.840	8.57	2.927	9.07	3.011	9.57	3.093
7.08	2.660	7.58	2.753	8.08	2.842	8.58	2.929	9.08	3.013	9.58	3.095
7.09	2.662	7.59	2.755	8.09	2.844	8.59	2.930	9.09	3.014	9.59	3.096
7.10	2.664	7.60	2.756	8.10	2.846	8.60	2.932	9.10	3.016	9.60	3.098
7.11	2.666	7.61	2.758	8.11	2.847	8.61	2.934	9.11	3.018	9.61	3.100
7.12	2.668	7.62	2.760	8.12	2.849	8.62	2.935	9.12	3.019	9.62	3.101
7.13	2.670	7.63	2.762	8.13	2.851	8.63	2.937	9.13	3.021	9.63	3.103
7.14	2.672	7.64	2.764	8.14	2.853	8.64	2.939	9.14	3.023	9.64	3.104
7.15	2.673	7.65	2.765	8.15	2.854	8.65	2.941	9.15	3.024	9.65	3.106
7.16	2.675	7.66	2.767	8.16	2.856	8.66	2.942	9.16	3.026	9.66	3.108
7.17	2.677	7.67	2.769	8.17	2.858	8.67	2.944	9.17	3.028	9.67	3.109
7.18	2.679	7.68	2.771	8.18	2.860	8.68	2.946	9.18	3.029	9.68	3.111
7.19	2.681	7.69	2.773	8.19	2.861	8.69	2.947	9.19	3.031	9.69	3.112
7.20	2.683	7.70	2.774	8.20	2.863	8.70	2.949	9.20	3.033	9.70	3.114
7.21	2.685	7.71	2.776	8.21	2.865	8.71	2.951	9.21	3.034	9.71	3.116
7.22	2.687	7.72	2.778	8.22	2.867	8.72	2.952	9.22	3.036	9.72	3.117
7.23	2.688	7.73	2.780	8.23	2.868	8.73	2.954	9.23	3.038	9.73	3.119
7.24	2.690	7.74	2.782	8.24	2.870	8.74	2.956	9.24	3.039	9.74	3.120
7.25	2.692	7.75	2.783	8.25	2.872	8.75	2.958	9.25	3.041	9.75	3.122
7.26	2.694	7.76	2.785	8.26	2.874	8.76	2.959	9.26	3.043	9.76	3.124
7.27	2.696	7.77	2.787	8.27	2.875	8.77	2.961	9.27	3.044	9.77	3.125
7.28	2.698	7.78	2.789	8.28	2.877	8.78	2.963	9.28	3.046	9.78	3.127
7.29	2.700	7.79	2.791	8.29	2.879	8.79	2.964	9.29	3.047	9.79	3.128
7.30	2.701	7.80	2.792	8.30	2.880	8.80	2.966	9.30	3.049	9.80	3.130
7.31	2.703	7.81	2.794	8.31	2.882	8.81	2.968	9.31	3.051	9.81	3.132
7.32	2.705	7.82	2.796	8.32	2.884	8.82	2.969	9.32	3.052	9.82	3.133
7.33	2.707	7.83	2.798	8.33	2.886	8.83	2.971	9.33	3.054	9.83	3.135
7.34	2.709	7.84	2.800	8.34	2.887	8.84	2.973	9.34	3.056	9.84	3.136
7.35	2.711	7.85	2.801	8.35	2.889	8.85	2.974	9.35	3.057	9.85	3.138
7.36	2.712	7.86	2.803	8.36	2.891	8.86	2.976	9.36	3.059	9.86	3.140
7.37	2.714	7.87	2.805	8.37	2.893	8.87	2.978	9.37	3.061	9.87	3.141
7.38	2.716	7.88	2.807	8.38	2.894	8.88	2.979	9.38	3.062	9.88	3.143
7.39	2.718	7.89	2.808	8.39	2.896	8.89	2.981	9.39	3.064	9.89	3.144
7.40	2.720	7.90	2.810	8.40	2.898	8.90	2.983	9.40	3.065	9.90	3.146
7.41	2.722	7.91	2.812	8.41	2.900	8.91	2.984	9.41	3.067	9.91	3.148
7.42	2.723	7.92	2.814	8.42	2.901	8.92	2.986	9.42	3.069	9.92	3.149
7.43	2.725	7.93	2.816	8.43	2.903	8.93	2.988	9.43	3.070	9.93	3.151
7.44	2.727	7.94	2.817	8.44	2.905	8.94	2.989	9.44	3.072	9.94	3.152
7.45	2.729	7.95	2.819	8.45	2.906	8.95	2.991	9.45	3.074	9.95	3.154
7.46	2.731	7.96	2.821	8.46	2.908	8.96	2.993	9.46	3.075	9.96	3.155
7.47	2.733	7.97	2.823	8.47	2.910	8.97	2.995	9.47	3.077	9.97	3.157
7.48	2.734	7.98	2.824	8.48	2.912	8.98	2.996	9.48	3.078	9.98	3.159
7.49	2.736	7.99	2.826	8.49	2.913	8.99	2.998	9.49	3.080	9.99	3.160
7.50	2.738	8.00	2.828	8.50	2.915	9.00	3.000	9.50	3.082	10.00	3.162

ANSWERS TO PROBLEMS

Chapter 9. Common Average: **1.** 75.4; **2.** 44.2; **3.** 112.5; **4.** 10.5; **5.** 35.3; **6.** 88.2; **7.** 72.1; **8.** 39.8; **9.** 56.2; **10.** 4.7; **11.** 151.9; **12.** 87.0; **13.** 66.8; **14.** 85.1; **15.** 101.1; **16.** 89.2; **17.** 63.1; **18.** 16.3; **19.** 104.0; **20.** 13.9; **21.** 15.6; **22.** 66.4; **23.** 4.5; **24.** 23.9; **25.** 104.5.

Chapter 10. Arithmetic Mean: **1.** 75.4; **2.** 121.0; **3.** 49.0; **4.** 24.0; **5.** 73.7; **6.** 27.5; **7.** 59.7; **8.** 72.4; **9.** 72.6; **10.** 94.0; **11.** 72.1; **12.** 43.9; **13.** 163.4; **14.** 22.0; **15.** 125.3; **16.** 69.1; **17.** 94.1; **18.** 57.7; **19.** 76.4; **20.** 72.1; **21.** 65.4; **22.** 74.3; **23.** 51.1; **24.** 74.4; **25.** 76.8; **26.** 30.6; **27.** 145.1; **28.** 72.6; **29.** 88.6; **30.** 26.8; **31.** 89.2; **32.** 126.6; **33.** 48.8; **34.** 70.8; **35.** 73.7; **36.** 122.7; **37.** 90.1; **38.** 65.8; **39.** 48.5; **40.** 75.0; **41.** 108.8; **42.** 118.0; **43.** 117.1; **44.** 65.3; **45.** 50.3; **46.** 62.2; **47.** 64.8; **48.** 271.1; **49.** 111.9; **50.** 76.6; **51.** 54.7; **52.** 58.0; **53.** 52.7; **54.** 69.6; **55.** 40.1.

Chapter 11. Mid-score: **1.** 89; **2.** 68; **3.** 45; **4.** 90; **5.** 16; **6.** 26; **7.** 47; **8.** 89; **9.** 67; **10.** 36; **11.** 92; **12.** 70; **13.** 110; **14.** 60; **15.** 72; **16.** 15; **17.** 83; **18.** 27; **19.** 14; **20.** 7; **21.** 61.5; **22.** 43; **23.** 134; **24.** 113; **25.** 70; **26.** 38; **27.** 88; **28.** 174; **29.** 204; **30.** 88; **31.** 17; **32.** 124; **33.** 16.4.

Chapter 12. Median: **1.** 72.5; **2.** 45.0; **3.** 58.5; **4.** 70.7; **5.** 62.4; **6.** 113.0; **7.** 58.0; **8.** 51.7; **9.** 87.5; **10.** 175.0; **11.** 92.4; **12.** 72.5; **13.** 44.1; **14.** 67.5; **15.** 79.4; **16.** 86.3; **17.** 59.0; **18.** 109.1; **19.** 81.1; **20.** 102.7; **21.** 95.8; **22.** 69.7; **23.** 83.5; **24.** 46.5; **25.** 167.4; **26.** 275.0; **27.** 27.1; **28.** 58.0; **29.** 58.5; **30.** 71.3; **31.** 83.4; **32.** 113.0; **33.** 62.4; **34.** 44.1; **35.** 45.8; **36.** 41.1; **37.** 40.4; **38.** 15.3; **39.** 89.9; **40.** 156.2; **41.** 73.0; **42.** 60.7; **43.** 54.7; **44.** 60.6;

45. 67.0; **46.** 93.9; **47.** 104.8; **48.** 62.3; **49.** 84.0; **50.** 116.8; **51.** 87.2; **52.** 51.1; **53.** 92.8; **54.** 101.1; **55.** 58.5; **56.** 54.5; **57.** 89.5; **58.** 110.7; **59.** 70.2; **60.** 57.2; **61.** 105.7; **62.** 58.8; **63.** 88.0; **64.** 44.2; **65.** 49.2; **66.** 137.8; **67.** 45.4; **68.** 45.0; **69.** 74.3; **70.** 42.0; **71.** 50.0; **72.** 56.0; **73.** 81.5; **74.** 146.0; **75.** 49.1; **76.** 46.7; **77.** 77.0; **78.** 95.8; **79.** 90.9; **80.** 46.8; **81.** 154.2; **82.** 59.6; **83.** 100.9; **84.** 22.7; **85.** 63.3.

Median by inspection: **1.** 55.0; **2.** 72.0; **3.** 45.0; **4.** 37.5; **5.** 75.0; **6.** 64.5; **7.** 84.0; **8.** 110.0; **9.** 67.5; **10.** 47.5; **11.** 60.0; **12.** 90.0; **13.** 92.0; **14.** 75.0; **15.** 55.0; **16.** 125.0; **17.** 70.0; **18.** 19.0; **19.** 87.5; **20.** 95.0; **21.** 19.0; **22.** 72.5; **23.** 109.0; **24.** 71.0; **25.** 52.5; **26.** 175.0; **27.** 43.5; **28.** 64.3; **29.** 180.0; **30.** 425.0; **31.** 52.5; **32.** 21.5; **33.** 61.0; **34.** 40.0; **35.** 72.5; **36.** 65.0; **37.** 32.0; **38.** 55.0; **39.** 46.0; **40.** 88.0; **41.** 67.5; **42.** 41.0; **43.** 104.0; **44.** 92.5; **45.** 75.7; **46.** 159.0; **47.** 70.0; **48.** 67.5; **49.** 62.9; **50.** 82.5; **51.** 109.0.

Chapter 13. Mode: **1.** 75; **2.** 45; **3.** 48, 50; **4.** 81, 90; **5.** 16; **6.** 8, 10; **7.** 125; **8.** 30; **9.** 87.5; **10.** 50, 75, 90; **11.** 30, 60, 90; **12.** 100.

Modal interval: **1.** 60–64; **2.** 60–69; **3.** 140–149; **4.** 60–64, 80–84; **5.** 30–31, 38–39; **6.** 66–68, 78–80; **7.** 140–144, 160–164, 170–174; **8.** 30–39; **9.** 65–69; **10.** 48–50; **11.** 120–124; **12.** 120–129; **13.** 70–74; **14.** 62–63, 72–73; **15.** 84, 87; **16.** 160–179; **17.** 15–17, 33–35; **18.** 30–39, 60–69.

Chapter 15. Average deviation, simple series: **1.** 6.0; **2.** 3.6; **3.** 5.1; **4.** 18.6; **5.** 5.4; **6.** 3.6; **7.** 2.8; **8.** 1.8; **9.** 4.2; **10.** 6.1; **11.** 5.2; **12.** 4.6; **13.** 3.2; **14.** 3.5; **15.** 7.1; **16.** 4.2; **17.** 6.6; **18.** 5.7; **19.** 5.7; **20.** 3.0; **21.** 5.5; **22.** 2.2; **23.** 2.5; **24.** 3.7; **25.** 4.0; **26.** 3.6; **27.** 10.8; **28.** 6.6; **29.** 10.0; **30.** 3.1; **31.** 2.0; **32.** 3.6; **33.** 5.5; **34.** 8.5; **35.** 3.8; **36.** 7.5; **37.** 1.3; **38.** 1.4; **39.** 1.2; **40.** 2.0; **41.** 3.0; **42.** 2.0; **43.** 1.5; **44.** 2.0.

Average deviation, frequency distribution: **1.** 14.6; **2.** 7.5; **3.** 28.8; **4.** 15.8; **5.** 8.5; **6.** 3.8; **7.** 5.1; **8.** 30.3; **9.** 17.6; **10.** 9.6; **11.** 3.0; **12.** 2.1; **13.** 6.5; **14.** 27.4; **15.** 8.8; **16.** 14.1; **17.** 12.4; **18.** 25.6; **19.** 9.1; **20.** 9.2; **21.** 16.3; **22.** 19.6; **23.** 8.3; **24.** 1.7; **25.** 16.8; **26.** 11.8; **27.** 1.7; **28.** 11.3; **29.** 22.3; **30.** 3.5; **31.** 5.9; **32.** 9.9; **33.** 9.4; **34.** 16.6; **35.** 38.1; **36.** 2.8.

Chapter 16. Standard deviation, simple series: **1.** 7.4; **2.** 2.6; **3.** 2.7; **4.** 2.9; **5.** 2.6; **6.** 1.7; **7.** 6.5; **8.** 12.0; **9.** 11.1; **10.** 7.4; **11.** 6.6; **12.** 7.0; **13.** 34.7; **14.** 5.3; **15.** 3.5; **16.** 10.9; **17.** 5.1; **18.** 4.7.

Standard deviation, frequency distribution: **1.** 18.6; **2.** 21.3; **3.** 9.9; **4.** 11.0; **5.** 10.8; **6.** 4.6; **7.** 12.6; **8.** 9.5; **9.** 23.0; **10.** 11.3; **11.** 11.7; **12.** 40.2; **13.** 18.1; **14.** 10.0; **15.** 34.8; **16.** 3.9; **17.** 1.9; **18.** 5.9; **19.** 45.0; **20.** 11.1; **21.** 6.0; **22.** 6.2; **23.** 5.3; **24.** 42.7; **25.** 39.6; **26.** 50.4; **27.** 12.1.

Chapter 17. Quartile deviation: **1.** 16.5; **2.** 8.7; **3.** 8.2; **4.** 18.8; **5.** 18.4; **6.** 12.6; **7.** 8.0; **8.** 50.0; **9.** 15.6; **10.** 3.0; **11.** 33.5; **12.** 8.6; **13.** 9.6; **14.** 29.6; **15.** 1.7; **16.** 5.0; **17.** 11.7; **18.** 9.0; **19.** 11.7; **20.** 8.5; **21.** 21.7; **22.** 24.7; **23.** 12.1; **24.** 33.7; **25.** 6.1; **26.** 8.1; **27.** 7.5; **28.** 24.2; **29.** 8.9; **30.** 2.4; **31.** 41.0; **32.** 8.1; **33.** 39.4; **34.** 6.7; **35.** 34.5; **36.** 24.6; **37.** 17.4; **38.** 9.9; **39.** 32.7; **40.** 16.2; **41.** 11.0; **42.** 13.0; **43.** 3.7; **44.** 8.9; **45.** 16.8; **46.** 12.2; **47.** 8.6; **48.** 3.9.

Chapter 19. Correlation (*rho*): **1.** +.63; **2.** +.75; **3.** +.70; **4.** +.39; **5.** −.52; **6.** +.68; **7.** +.86; **8.** +1.00; **9.** +.86; **10.** +.83; **11.** +.81; **12.** +.62; **13.** +.73; **14.** +.89; **15.** +.82; **16.** +.83; **17.** +.74; **18.** +.79; **19.** +.25; **20.** +1.00; **21.** −.05; **22.** +.69; **23.** −.72; **24.** +.85; **25.** +.87; **26.** +.94; **27.** −1.00; **28.** +.80; **29.** +.39; **30.** +.35; **31.** +.79; **32.** −.76; **33.** +.80.

Chapter 20. Correlation (r): **1.** +.27; **2.** +.80; **3.** +.67; **4.** +.35; **5.** +.37; **6.** +.51; **7.** +.77; **8.** +.77; **9.** +.64; **10.** −.06; **11.** +.21; **12.** +.54.

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